



Physics

Student Textbook Grade 10

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Section	Learning competencies
1.1 Projectile motion (page 2)	- Define the term projectile (and provide several examples). - Explain the difference between 1D and 2D motion. - Correctly use the terms angle of elevation and angle of depression, and explain the importance of the angle when it comes to launching projectiles. - Explain the effect gravity has on the motion of an object. - Describe what happens to the horizontal and vertical velocities of a projectile and the important characteristics of its flight. - Demonstrate how to use the equations for uniform acceleration and to apply these to projectile motion. - Define the term centre of mass. - Conduct simple experiments to determine the centre of mass of 2D objects. - List the characteristics of uniform circular motion. - Describe the relationships between radius, mass, forces and velocity for an object following a circular path.
1.2 Rotational kinematics (page 24)	- Define the terms angular and tangential displacement, and angular and tangential velocity. - Express angles in terms of revolutions, radians and degrees. - Define the term angular acceleration, and list its key characteristics. - Identify the SI unit of angular velocity and angular acceleration. - Explain the relationships between angular displacement, tangential displacement, angular velocity, tangential velocity and angular acceleration. - Demonstrate how to use the equations of constant angular acceleration and compare them with equations of constant acceleration.
1.3 Rotational dynamics (page 30)	- Define the moment of inertia of a point mass. - Define rotational kinetic energy of a body. - Solve simple problems relating to moment of inertia and rotational kinetic energy. - Define the term torque. - Identify the SI unit of torque, N m, which is not the same as the joule. - Express torque in terms of moment of inertia and angular acceleration. - Derive an expression for the work done by the torque. - Use the formula $W = \tau\theta$ to solve problems related to work done by torque.

Contents	
Section	Learning competencies
1.3 Rotational dynamics (page 30) (continued)	- Define the angular momentum of a particle of mass m and write its SI unit. - State the law of conservation of angular momentum. - Solve problems using the law of conservation of angular momentum. - State the first and second conditions of equilibrium. - Solve problems related to conditions of equilibrium. - Define the term centre of mass (centre of gravity) of a solid body. - Determine the centre of gravity using a plumb-line method. - Define the terms stable, unstable and neutral equilibrium.
1.4 Newton's law of universal gravitation (page 40)	- State Newton's law of universal gravitation. - Determine the magnitude of the force of attraction between two masses separated by a distance r. - Calculate the value of g at any distance above the surface of the Earth. - State Kepler's law of planetary motion. - Use Kepler's law of planetary motion to determine the period of any planet. - Differentiate between orbital and escape velocity of a satellite. - Determine the period of a satellite around a planet. - Calculate the orbital and escape velocity of a satellite. - Describe the period, position and function of a geostationary satellite.

You may have already studied motion in one dimension (1D), including the equation of uniform acceleration. This unit takes motion further, looking into motion in two dimensions. This includes projectiles and circular motion.

When you catch a ball your brain is completing a series of complex calculations relating to the path followed by the ball and the time it takes to reach you. This is hard-wired into our brains from the days when we used to hunt and most of our food lived in trees. A detailed understanding of two-dimensional (2D) motion is essential for physicists, as it enables them to complete the calculations required to design objects from complex rockets to roads around cities.

1.1 Projectile motion

By the end of this section you should be able to:

- Define the term projectile (and provide several examples).
- Explain the difference between 1D and 2D motion.
- Correctly use the terms angle of elevation and angle of depression, and explain the importance of the angle when it comes to launching projectiles.

- Explain the effect gravity has on the motion of an object.
- Describe what happens to the horizontal and vertical velocities of a projectile and the important characteristics of its flight.
- Demonstrate how to use the equations for uniform acceleration and to apply these to projectile motion.
- Define the term centre of mass.
- Conduct simple experiments to determine the centre of mass of 2D objects.
- List the characteristics of uniform circular motion.
- Describe the relationships between radius, mass, forces and velocity for an object following a circular path.

What are projectiles?

A **projectile** is any object moving through the air without an engine or other motive force. This means it is not restricted to cannonballs or bullets. When you throw a stone, toss a cricket ball, or kick a soccer ball, they are classed as projectiles as they fly through the air.

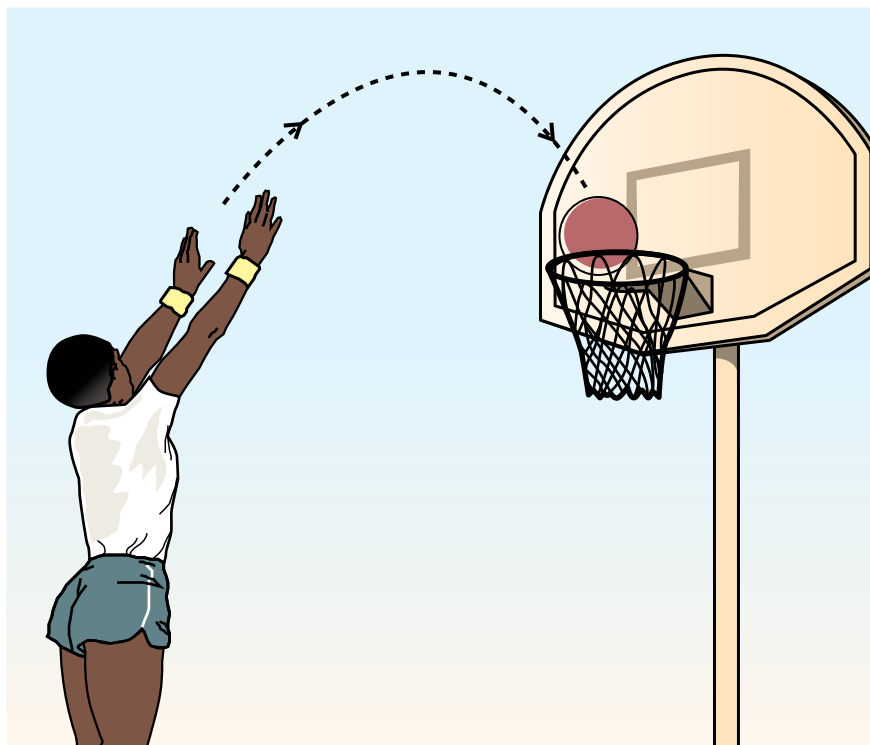


Figure 1.1 Typical example of a projectile.

Projectile motion is more complex than 1D motion. It involves motion in two directions, the vertical and the horizontal.

For simplicity we look at the horizontal **velocity** and vertical velocity as separate components of the velocity. These can be treated individually.

KEY WORDS

projectile any object propelled through space by the exertion of a force which ceases after launch

velocity the rate of change of position of a body

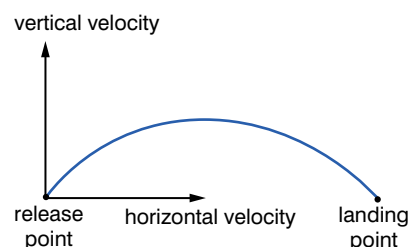


Figure 1.2 Another projectile.

KEY WORDS

resolving *splitting a vector into vertical and horizontal components. These components have the same effect as the original vector.*

Activity 1.1: Naming some projectiles

Spend 30 seconds listing as many projectiles as you can think of. Compare your list with a partner.

Resolving velocity

Resolving means splitting one vector into two component vectors (usually one horizontal and one vertical). These components have the same effect as the original vector.

An example can be seen below. The 60 m/s velocity can be resolved into two component vectors that have the same effect when combined:

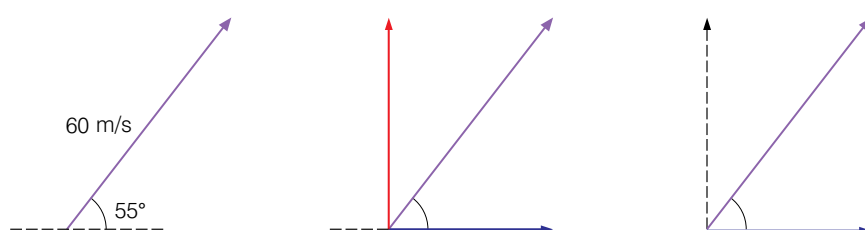


Figure 1.3 Velocity components are shown in red and blue.

This allows us to calculate the path of the projectile including its maximum vertical displacement (maximum height), maximum horizontal displacement (range) and flight time.

Horizontal motion

If we ignore air resistance, then there are no horizontal forces acting on the projectile as it flies through the air. This means there is no acceleration, so the velocity stays the same horizontally. This is a perfectly valid assumption for many projectiles.

We can apply the following equation:

$$\text{displacement} = \text{average velocity} \times \text{time taken}$$

This becomes:

$$\text{horizontal displacement} = \text{horizontal velocity} \times \text{flight time}$$

So in order to determine the range of the projectile we would use the total flight time. For example, to find the horizontal displacement after 5.0 s we would use a time of 5.0 seconds.

$$\text{total horizontal displacement} = \text{horizontal velocity} \times \text{total flight time}$$

$$\text{horizontal displacement after 5.0 s} = \text{velocity} \times 5.0 \text{ s}$$

Discussion activity

Under what circumstances might it be inappropriate to assume the horizontal velocity of a projectile remains constant?

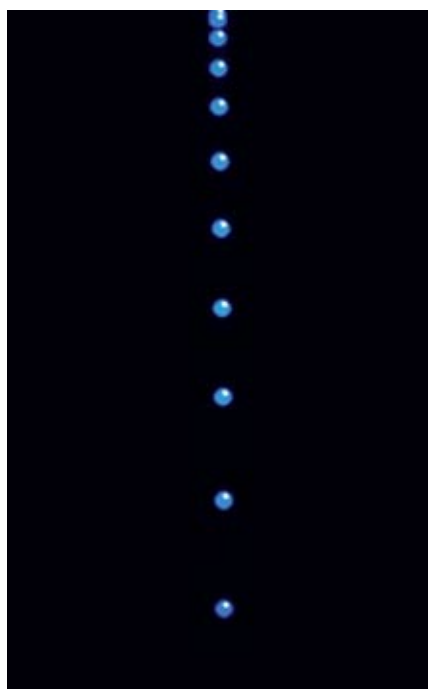


Figure 1.4 Accelerating under gravity.

Vertical motion

The vertical velocity does change. This is because the projectile accelerates under gravity as it moves.

Figure 1.4 shows a ball falling through the air. The images were taken at regular time intervals and you can see the displacement between each shot increases. This is because the ball is moving faster and faster.

Ignoring air resistance once again, gravity causes all objects to accelerate at 9.81 m/s^2 .

Figure 1.5 shows two motion graphs for the falling ball. Looking at the displacement–time graph it is evident the gradient is increasing. This is because the object is moving faster as it falls.

This can be seen on the velocity–time graph. A constant gradient indicates a constant acceleration (in this case, 9.81 m/s^2).

The vertical motion of the projectile is an example of uniformly accelerated motion. This means we can use the equations for uniform acceleration:

1. $v = u + at$
2. $s = \frac{1}{2}(u + v)t$
3. $s = ut + \frac{1}{2}at^2$
4. $v^2 = u^2 + 2as$
5. $s = vt - \frac{1}{2}at^2$

where:

s = displacement

v = final velocity

u = initial velocity

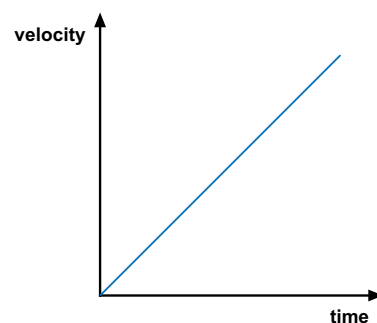
a = acceleration (in this case, 9.81 m/s^2)

t = time

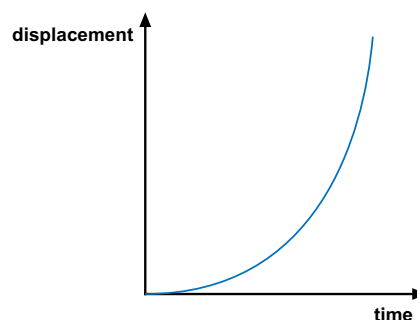
These can be used to determine the time it takes for a projectile to hit the ground.

DID YOU KNOW?

Due to the shape of the Earth and its uneven density, the value of the acceleration due to gravity varies slightly. At the Equator it is 9.78 m/s^2 whereas at the North Pole it is 9.83 m/s^2 . 9.81 m/s^2 is usually referred to as Standard Gravity.



a



b

Figure 1.5 Graphs showing the vertical velocity and displacement as the ball falls.

Worked example 1.1

Find the time taken for a ball dropped from a height of 6.0 metres:

s (m)	u (m/s)	v (m/s)	a (m/s ²)	t (s)
6.0	0.0 (as dropped)	unknown	9.81	?

We don't know the final velocity so we must use equation 3 (because there is no v in this equation).

DID YOU KNOW?

If we ignore air resistance, the mass of an object does not affect the rate at which it accelerates. Galileo Galilei was the first to realise this back in the 17th century. At the end of the last Apollo 15 moon walk, Commander David Scott tested this theory. He dropped a hammer and a feather at the same time. As the surface of the moon is a vacuum, there was no air resistance and the feather fell at the same rate as the hammer.

$$s = ut + \frac{1}{2}at^2$$

$ut = 0$ as the ball was dropped so the equation becomes

$$s = \frac{1}{2}at^2$$

This can be rearranged to $t = \sqrt{\frac{2s}{a}}$

$$t = \sqrt{\frac{2 \times 6.0 \text{ m}}{9.81 \text{ m/s}^2}}$$

$$t = 1.1 \text{ s (to two significant figures)}$$

Activity 1.2: Dropping a ball

Drop a ball from several different heights and time how long it takes to hit the ground. Record your data carefully and take repeat measurements for each height.

Using the equation, calculate the time it actually takes to hit the ground. Compare the actual times with your readings and comment on your findings.



Figure 1.6 Galileo Galilei.

We can also work out the final vertical velocity.

Worked example 1.2

Find the final vertical velocity for a ball dropped from 6 m. Looking back at the table we now have:

s (m)	u (m/s)	v (m/s)	a (m/s ²)	t (s)
6.0	0.0 (as dropped)	unknown	9.81	1.1

We could use equations 1, 2, 4, or 5 to determine v . However, equation 4 does not require you to calculate time, so this is preferable.

$$v^2 = u^2 + 2as$$

$$v = \sqrt{u^2 + 2as}$$

$$v = \sqrt{0^2 + 2 \times 9.81 \text{ m/s}^2 \times 6.0 \text{ m}}$$

$$v = 11 \text{ m/s (to two significant figures)}$$

KEY WORDS

parabola the curved path a projectile takes through the air

How does this affect the motion of projectiles?

When a projectile moves through the air, it follows a path caused by the combination of its horizontal and vertical velocities. This path is curved; it forms a special type of curve called a **parabola**.

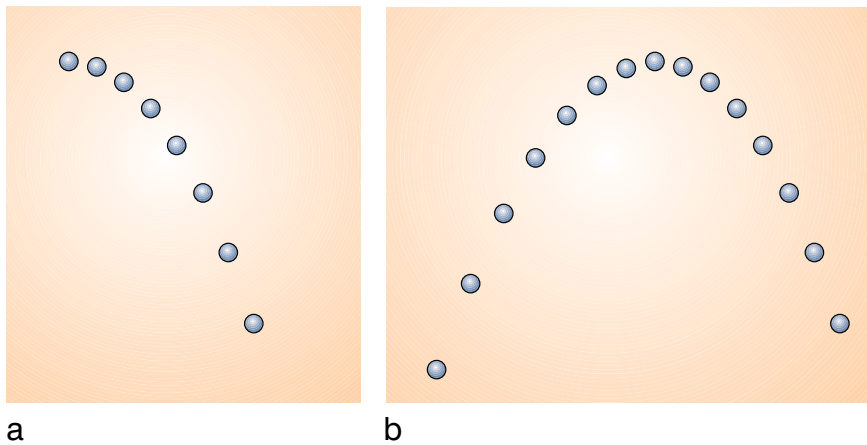


Figure 1.7 Diagrams of paths of balls.

Figure 1.9 shows a real example of this type of motion. Here one child throws a ball to another. A camera took photos at regular time intervals. If you look carefully, you can see the ball's vertical velocity increases as it falls and decreases as it rises.

Notice:

- Horizontally, the ball moves at a steady speed. The images of each ball are equally spaced horizontally.
- Vertically, the ball accelerates downwards due to gravity. This means the images become further and further apart.

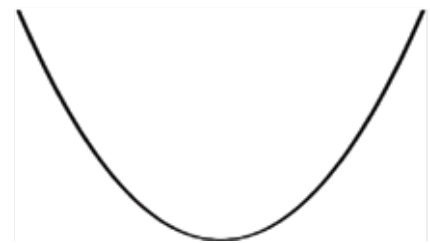


Figure 1.8 A parabola.



Figure 1.9 A multi-flash image showing the motion of a projectile (a ball).

Horizontal and vertical velocities

This special shape is caused by the relationship between the horizontal and vertical velocities.

Activity 1.3: The path of a ball

Next time you are outside, get two friends to throw a ball to each other. Stand at the side (some distance away) and watch the path of the ball carefully. Try asking your friends to throw the ball at different angles. What do you notice?

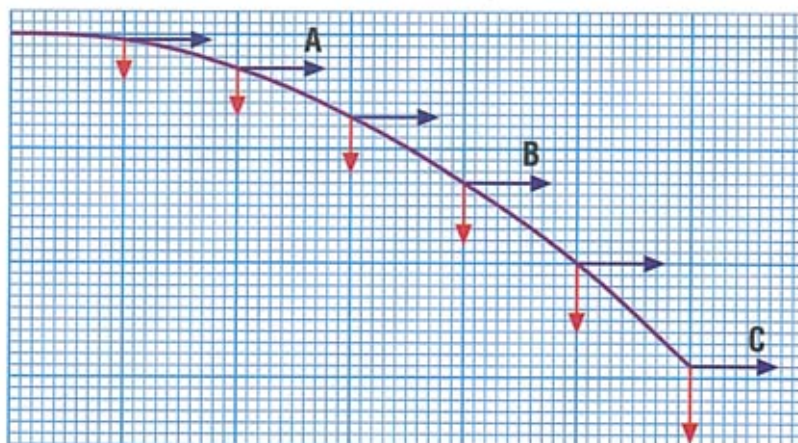


Figure 1.10 Horizontal and vertical velocities.

If you look carefully at Figure 1.10 you can see the horizontal velocity remains the same but the vertical velocity increases. This causes the ball to follow a parabolic path.

The same is true for a projectile thrown at an angle.

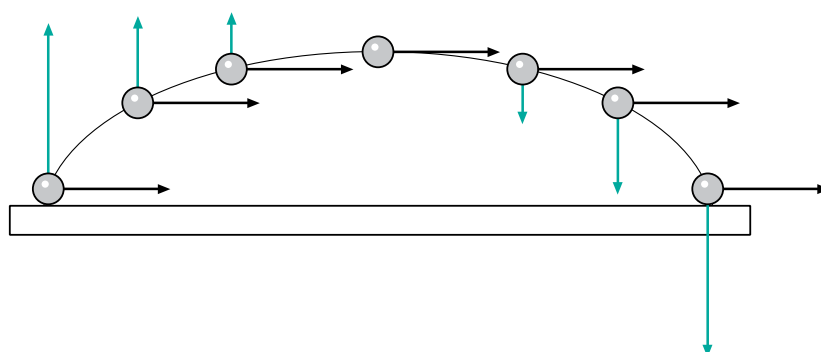


Figure 1.11 Horizontal and vertical velocities for a projectile thrown at an angle.

Figure 1.11 shows the velocities for a ball thrown at an angle. Again the horizontal velocity remains constant. The vertical velocity first decreases (on the way up), reaches zero (at the top of the flight), and then increases on the way down. The vertical velocity at any point is given by the equation $v = u + at$.

You need to think carefully about the directions of the vertical velocities and the acceleration. If we use a as 9.81 m/s^2 , then the initial vertical velocity must be a negative number as it is in the opposite direction. We have effectively decided that downwards is the positive direction.

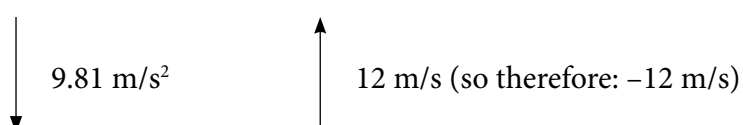


Figure 1.12 The importance of direction when dealing with vectors.

This works the other way around, too. It does not matter which way is positive and which way is negative, but you must not mix them up!

Worked example 1.3

An arrow is fired vertically with an initial velocity of 35 m/s. Find its velocity after:

a) 3 s

b) 6 s

Using the table layout seen earlier we get:

s (m)	u (m/s)	v (m/s)	a (m/s ²)	t (s)
Unknown	-35	?	9.81	a) 3 b) 6

Notice we have entered -35 m/s for the initial velocity. We are therefore setting the downwards direction as positive.

a)

$$v = u + at$$

$$v = -35 \text{ m/s} + (9.81 \text{ m/s}^2 \times 3 \text{ s})$$

$$v = -5.6 \text{ m/s (to 2 significant figures)}$$

Notice the velocity is still negative as it is still travelling upwards.

b)

$$v = u + at$$

$$v = -35 \text{ m/s} + (9.81 \text{ m/s}^2 \times 6 \text{ s})$$

$$v = 24 \text{ m/s (to 2 significant figures)}$$

Notice the velocity is now positive. This must mean the arrow has changed direction and is heading back down.

Activity 1.4: Heights of arrows

Calculate the height of the arrow in each case for the worked example.

Horizontal projection

Projectiles may be initially travelling horizontally. This might include a ball kicked off a wall, a bullet fired from a horizontal gun, or a parcel dropped from the underside of an aircraft flying horizontally.

The object will follow the path shown in Figure 1.13. It is interesting to note that the time it takes to hit the floor only depends on the original height of the object.

The flight time is given by the equation $s = ut + \frac{1}{2}at^2$. Looking at this vertically, $ut = 0$ as the ball initially has no vertical velocity. So the equation becomes $s = \frac{1}{2}at^2$ and the time it takes to hit the floor is given by:

$$t = \sqrt{\frac{2s}{a}}$$

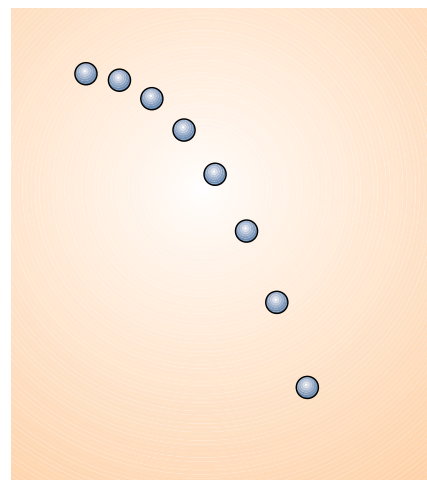


Figure 1.13 Horizontal projection.

The acceleration is constant (9.81 m/s^2) and so the only factor affecting the time to hit the floor is the drop height.

From this we can draw some counterintuitive conclusions: imagine someone holding a rifle in their right hand and a rifle bullet in their left hand. The rifle is perfectly horizontal. The rifle is then fired and the bullet is dropped at exactly the same time. Which bullet will hit the ground first?

It would be tempting to say the bullet from the rifle takes longer to hit the ground than the one dropped from the hand. However, this would be wrong! They both hit the ground at the same time; the only difference is the bullet from the rifle hits the ground several metres away whereas the dropped bullet hits the ground by the person's feet.

Figure 1.14 shows a time lapse of two balls: one dropped, the other fired horizontally. Both were released simultaneously. You can clearly see they stay at exactly the same height as they fall.

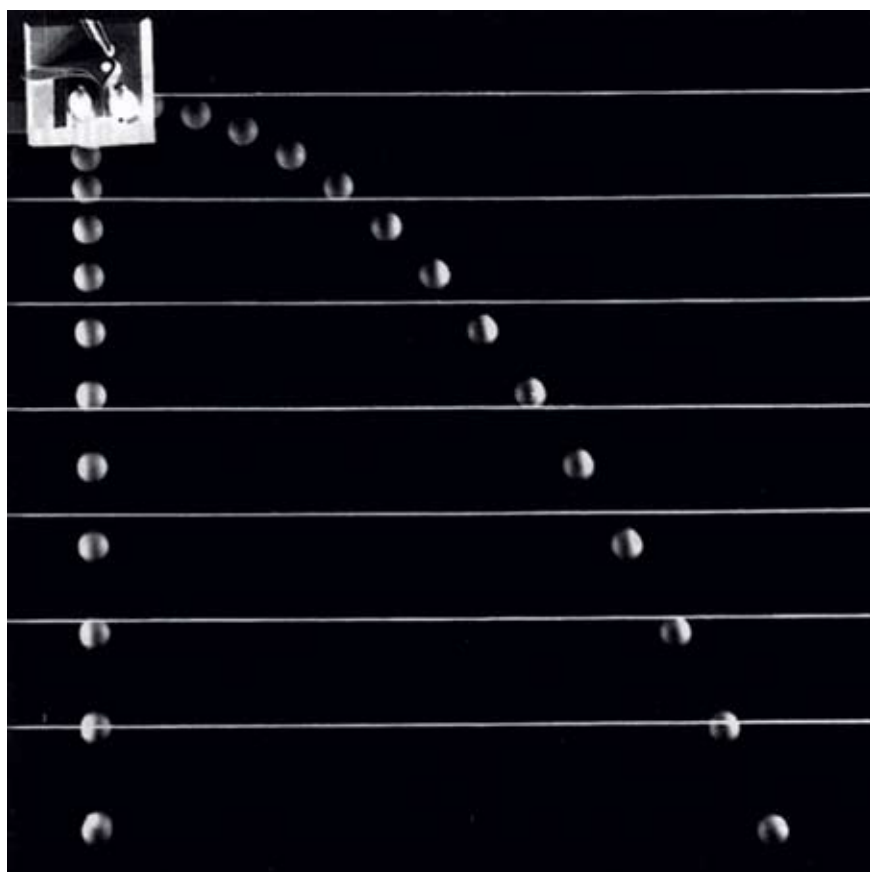


Figure 1.14 Time lapse of two balls falling.

Activity 1.5: Test with ruler and two coins

You can test this using a ruler and two coins. Carefully place the ruler on the edge of a desk. Place one coin on the end of the ruler overhanging the desk. Place the other next to the ruler as shown. The aim is to flick the ruler so that the first coin falls vertically whereas the second coin gets pushed off the desk horizontally.

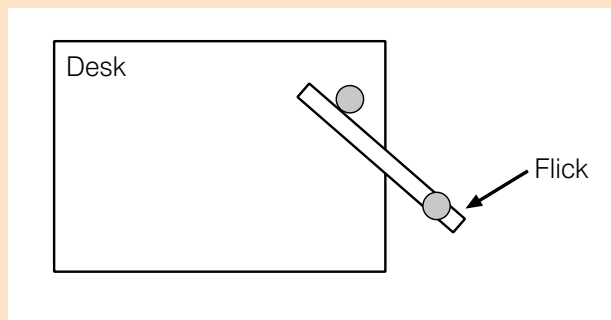


Figure 1.16 Ruler and coin experiment.

You have to flick it quite hard.

When you do so, listen for the clink as the coins hit the floor. You will find the two clinks come at once; both coins hit the floor at the same time.

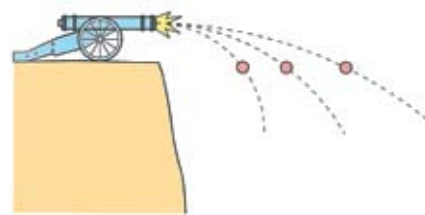


Figure 1.15 Cannonballs fired horizontally with different velocities.

Activity 1.6: Rolling a ball down a track

A more complex experiment involves rolling a ball or marble down a track. As you vary the release height, you vary the horizontal velocity of the marble. You could time the time it takes to hit the ground for different release heights.

You must be careful to start timing when the ball leaves the desk.

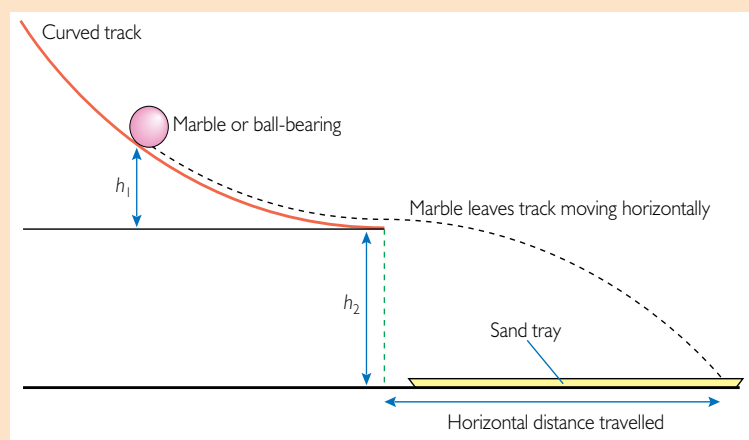


Figure 1.17 A marble rolling down a track.

Worked example 1.4

A football is kicked off a wall at an initial horizontal velocity of 12 m/s. The wall is 2.1 m high. Find the time taken for the ball to hit the floor and the range of the ball.

s (m)	u (m/s)	v (m/s)	a (m/s ²)	t (s)
2.1	12	?	9.81	?

$$t = \sqrt{\frac{2s}{a}}$$

$$t = \sqrt{\frac{2 \times 2.1 \text{ m}}{9.81 \text{ m/s}^2}}$$

$$t = 0.65 \text{ s}$$

horizontal displacement =
horizontal velocity $\times$ flight
time

$$\text{horizontal displacement} = 12 \text{ m/s} \times 0.65 \text{ s}$$

$$\text{horizontal displacement} = 7.8 \text{ m}$$

As discussed earlier, the range of any projectile is given by:

horizontal displacement = horizontal velocity (v_h) $\times$ flight time

The flight time is given by: $t = \sqrt{\frac{2s}{a}}$

Combining these we get:

$$\text{horizontal displacement} = v_h \times \sqrt{\frac{2s}{a}}$$

$$\text{horizontal displacement} = v_h \times \sqrt{\frac{2 \text{ vertical height}}{\text{acceleration}}}$$

This only applies to projectiles initially travelling horizontally.

Activity 1.7: Trajectory of a ball

Use the equations above to complete the table below. Use this data and a piece of graph paper to carefully plot the trajectory of a ball thrown horizontally with a velocity of 2.0 m/s.

Time (s)	Vertical displacement y (m)	Horizontal displacement x (m)
1.0		
2.0		
3.0		
4.0		
5.0		

Activity 1.8: Range of a ball rolled down a track

Using the same equipment as for Activity 1.6, roll the marble down the track. This time, measure the range of the marble when released from several different heights up the ramp. Be sure to repeat your readings for each height.

Plot a graph of your findings and use your data to calculate the initial velocity in each case.

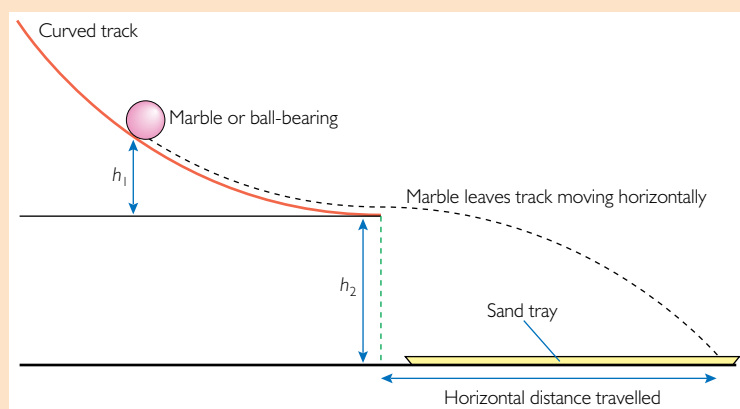


Figure 1.18 A marble rolling down a track.

Projectiles at angles

Projectiles may also be travelling at angles to the horizontal. In this case their initial vertical velocity is not zero. In order to find the initial horizontal and initial vertical velocity, the velocity must be resolved into horizontal and vertical components.

For example, a ball kicked with a velocity of 9.0 m/s at an angle of 50° to the horizontal:

To find the components, we use trigonometry.

- initial vertical component = 9.0 m/s $\times$ sin 50°
- initial vertical component = 6.9 m/s
- initial horizontal component = 9.0 m/s $\times$ cos 50°
- initial horizontal component = 5.8 m/s (remember this will not change throughout the flight of the projectile).

If the angle of the projectile is above the horizontal, then this is referred to as an **angle of elevation** (Figure 1.20a). However, if the angle of the projectile is below the horizontal then this is referred to as an **angle of depression** (Figure 1.20b).

Using the initial vertical velocity, the initial horizontal velocity and the equations of uniform acceleration we can then determine the range, flight time and maximum height of the projectile.

Maximum height

At the maximum height the vertical velocity of the projectile will be zero. Using the example in Figure 1.19 (9.0 m/s at 50°) we can use the equations of uniform acceleration as follows:

s (m)	u (m/s)	v (m/s)	a (m/s ²)	t (s)
??	$u \sin \theta$ in this case: -6.9	At max height: 0	9.81	unknown

Again, take care to ensure you consider the directions of the velocities; in this case, downwards is positive.

To find the maximum height we use $v^2 = u^2 + 2as$ (as we don't know t):

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{(0^2 - (-6.9^2 \text{ m/s}^2))}{2 \times 9.81 \text{ m/s}^2}$$

$$= -2.4 \text{ m.}$$

The height is of course 2.4 m, the negative just indicates that it is in the opposite direction to the acceleration.

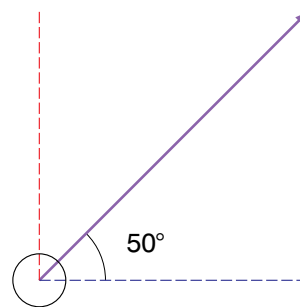


Figure 1.19 Ball kicked at an angle.

KEY WORDS

angle of elevation the angle of a projectile's trajectory above the horizontal

angle of depression the angle of a projectile's trajectory below the horizontal

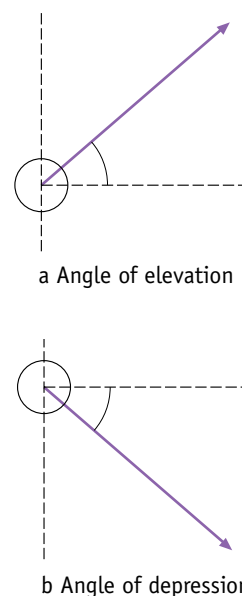


Figure 1.20 Angle of elevation and angle of depression.

We can then work out the time taken to reach maximum height using $v = u + at$.

$$v = u + at$$

$$t = \frac{v - u}{a}$$

$$t = \frac{0 - -6.9 \text{ m/s}}{9.81 \text{ m/s}^2}$$

$$t = 0.70 \text{ s}$$

Projectile range

If the projectile is launched from the ground, the total range will be given by:

- Total horizontal displacement = horizontal velocity $\times$ total flight time

The horizontal velocity remains the same throughout the flight and is given by $u \cos \theta$ (in the previous example, 5.8 m/s). All we need to find the range is total flight time.

This can be done in a number of ways; perhaps the simplest is to use $v = u + at$. The final vertical velocity will be the same magnitude as the initial vertical velocity, just in the opposite direction (look at Figure 1.11 to check).

Using the same example as before we get:

s (m)	u (m/s)	v (m/s)	a (m/s ²)	t (s)
??	$u \sin \theta$ in this case: -6.9	6.9	9.81	unknown

Again, take care to ensure you consider the directions of the velocities; in this case downwards is positive. Notice the final velocity is the same magnitude as the initial velocity, just the opposite direction (hence positive and not negative).

We can then work out the time taken to reach maximum height using $v = u + at$.

$$v = u + at$$

$$t = \frac{v - u}{a}$$

$$t = \frac{6.9 \text{ m/s} - -6.9 \text{ m/s}}{9.81 \text{ m/s}^2}$$

$$t = 1.4 \text{ s}$$

The total flight time is just double the time to reach maximum height!

So the range of the football is:

Total horizontal displacement = horizontal velocity $\times$ total flight time

$$\text{Total horizontal displacement} = 5.8 \text{ m/s} \times 1.4 \text{ s}$$

$$\text{Total horizontal displacement} = 8.1 \text{ m}$$

Alternatively, you can use algebra to combine the equation above, the horizontal and vertical components of the velocity, and the equations of uniform motion. This gives an expression for the range as:

$$\text{range} = \frac{u \cos \theta \times 2u \sin \theta}{a}$$

or

$$\text{range} = \frac{2u^2 \sin \theta \times \cos \theta}{a} = \frac{u^2 \sin 2\theta}{a}$$

This is usually called the **range equation**.

Activity 1.9

In a small group, discuss why

$$\frac{2u^2 \sin \theta \times \cos \theta}{a} = \frac{u^2 \sin 2\theta}{a}$$

Worked example 1.5

Find the range of a projectile launched at an angle of 50° with an initial velocity of 30 m/s.

range (m)	u (m/s)	a (m/s ²)	θ (°)	2θ (°)	$\sin 2\theta$
?	30	9.81	50°	100°	0.9848

$$\text{Use range} = \frac{u^2 \sin 2\theta}{a}$$

$$\begin{aligned} \text{range} &= \frac{30 \times 30 \times 0.9848}{9.81} \\ &= 90.35 \text{ m} \end{aligned}$$

KEY WORDS

range equation an algebraic expression to calculate the range of a projectile

Flight time

You can also derive an equation for the total flight time for a projectile fired at an angle. This is just a version of $s = vt - \frac{1}{2}at^2$ (equation 5 in our list of equations for constant acceleration).

At the end of the flight the vertical displacement will be 0 m, as the object will be back on the ground. The final vertical velocity will be given by $u \sin \theta$, as the object ends up with the same vertical velocity as it started, just in the opposite direction. So:

$$s = vt - \frac{1}{2}at^2$$

$$0 = (u \sin \theta) t - \frac{1}{2}at^2$$

Rearranging this we get:

$$\frac{1}{2} a t^2 = (u \sin \theta) t$$

$$\frac{1}{2} a t = u \sin \theta$$

$$a t = 2 u \sin \theta$$

$$t = \frac{2u \sin \theta}{a}$$

Worked example 1.6

Find the flight time for a cannonball launched with a velocity of 60 m/s at an angle of 30°.

t (s)	u (m/s)	θ (°)	$\sin \theta$ (°)	a (m/s ²)
?	60	30	0.5	9.81

$$t = \frac{2u \sin \theta}{a}$$

$$t = \frac{2 \times 60 \sin 30}{9.81}$$

$$t = 6.1 \text{ s}$$

Its range would be:

$$\text{Range} = \frac{2u^2 \sin \theta \times \cos \theta}{a}$$

$$\text{Range} = \frac{2 \times 60^2 \sin 30 \times \cos 30}{9.81}$$

$$\text{Range} = 320 \text{ m}$$

Remember that all of these equations ignore air resistance. The equations become exceptionally complex when this is factored in, especially since the air resistance changes as the velocity changes.

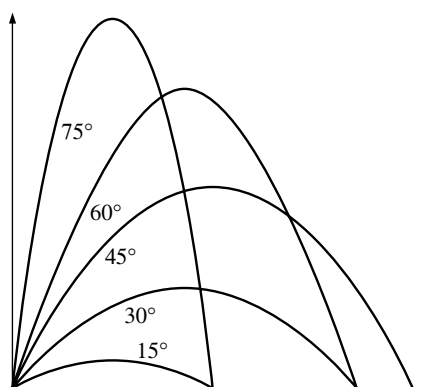


Figure 1.21 An illustration of the effect of angle on range.

Maximum range

The angle of a projectile affects its maximum range.

Figure 1.21 shows different paths of a projectile fired at different angles. The maximum range is achieved when the angle is 45°. This can be explained by referring back to the range equation:

$$\text{Range} = \frac{2u^2 \sin \theta \times \cos \theta}{a}$$

The maximum value of $\sin \theta \times \cos \theta$ is 0.5 and this happens when θ is 45°. This gives us:

$$\text{maximum range} = \frac{2u^2 \cdot 0.5}{a}$$

$$\text{maximum range} = \frac{u^2}{a}$$

To determine the range of projectiles launched above the ground you need to use algebra to derive a new equation. The range is given by:

$$\text{Range} = \frac{(u \cos \theta) \times (u \sin \theta + \sqrt{(u \sin \theta)^2 + 2ah})}{a}$$

where h is the height above the ground. Even this equation ignores air resistance!

Activity 1.10: The range of a cannonball

Use the range equation to determine the range of a cannonball fired with a velocity of 50 m/s when fired at a series of different angles. Use 15° , 30° , 45° , 60° and 75° . Plot a graph of range against angle and comment on your findings.

Worked example 1.7

Use this equation to determine the range of the ball in Figure 1.22.

u (m/s)	θ ($^\circ$)	$\cos \theta$ ($^\circ$)	$\sin \theta$	a (m/s ²)	h (m)	range (m)
36	39	0.7771	0.6293	9.81	1.6	?

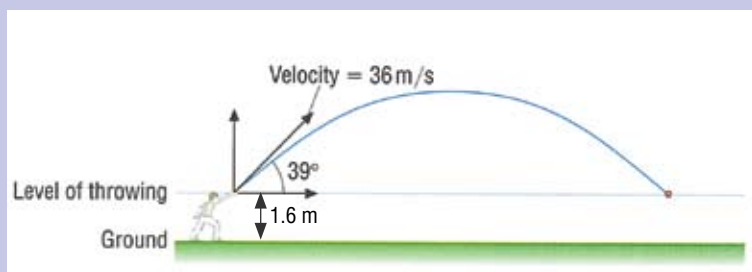


Figure 1.22 Throwing a soccer ball.

$$\begin{aligned}
 \text{Use range} &= \frac{(u \cos \theta) \times (u \sin \theta + \sqrt{(u \sin \theta)^2 + 2ah})}{a} \\
 &= \frac{(36 \times 0.7771) \times (36 \times 0.6293 + \sqrt{(36 \times 0.6293)^2 + 2 \times 9.81 \times 1.6})}{9.81} \\
 &= \frac{27.9756 \times (22.6548 + \sqrt{(22.6548)^2 + 31.392})}{9.81} \\
 &= \frac{27.9756 \times (22.6548 + \sqrt{513.24 + 31.392})}{9.81} \\
 &= \frac{27.9756 \times (22.6548 + \sqrt{544.632})}{9.81} \\
 &= \frac{27.9756 \times (22.6548 + 23.3374)}{9.81} \\
 &= \frac{27.9756 \times 45.9922}{9.81} = \frac{1286.66}{9.81} = 131.16 \text{ m}
 \end{aligned}$$

KEY WORDS

centre of mass *the point at which all the mass of an object may be considered to be concentrated*

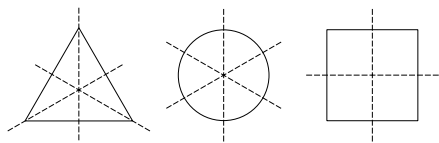


Figure 1.23 Examples of the centre of mass for different uniform shapes.

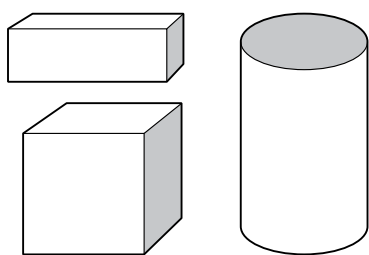


Figure 1.24 The centre of mass for different uniform 3D objects is in the centre of the object.

Centre of mass

All objects have a **centre of mass**. This is the point at which all the mass of the object may be considered to be concentrated.

For uniform objects the centre of mass will be at the intersection of all the lines of symmetry; in essence the centre of the object.

This is also true of 3D objects.

Another way to define the centre of mass is: the point through which a single force on a body has no turning effect.

Activity 1.11: Centre of mass of a ruler

Take a ruler and balance it on your finger. When balanced, the centre of mass must be above your finger. Take several other objects and balance them on your finger to find the approximate position of the centre of mass. Try it with a few more irregularly shaped objects.

This idea of balancing an object so there is no net turning effect leads onto the centre of mass theorem. This is a mathematical treatment of the distribution of mass, which is beyond the scope of this course.

The centre of mass theorem

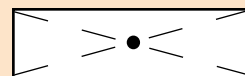
The centre of mass theorem simply states:

- when a force is applied to an object, the object acts as though its mass were a point mass at its centre of mass.

This means that the motion of the centre of mass of a system is identical to the motion of a single particle with the same mass as the system if such a particle were acted on by the same external forces.

Activity 1.12: Motion of the centre of mass

Work in a small group. You need a 2 m plank with four low friction wheels (for example, roller bearings).



- Mark the middle of the plank as shown in the diagram (this is the centre of mass), on the plank and on the floor beneath.
- One person should stand at the middle of the plank and walk four steps.
- Mark the position where the student finishes on the plank and on the floor.
- Record the distance that the centre of mass has moved.
- Use your knowledge of motion to analyse the motion of the centre of mass.

The centre of mass where two or more celestial bodies orbit each other is known as the barycentre. This is the point between the bodies where they balance each other. The Moon does not orbit

the exact centre of the Earth, but their masses balance at a point approximately 1710 km below the surface of the Earth on a line between the Earth and the Moon.

Experimental determination of centre of mass

It is quite difficult to accurately determine the centre of mass for a 3D object. Special machines called planimeters are used. However, it is quite simple to determine the mass of a 2D object.

Activity 1.13: Centre of mass of an object

1. Take a piece of thick card and cut it into any shape you like (this example involves using a piece shaped like a jigsaw piece).
2. Make a series of small holes around the edge of the shape.
3. Hang it from one of these holes so the object is free to rotate.
4. Construct a simple plumb line using some wire (or string) and a mass.
5. Hang this from the support so it hangs vertically down.
6. Using a sharp pencil, draw a line to show the position of the plumb line.
7. Repeat this procedure for all the holes you have made, making a series of straight lines on your shape.
8. The lines should all cross; this is the centre of mass of the object. (You can test this by balancing the shape on the sharp pencil.)

The centre of mass of a system does not have to be an object. Take, for example, a cup. The centre of mass will be inside the cup even though there is nothing there but air. A more complex example might be two binary stars; these orbit the centre of mass between the two stars.

Uniform circular motion

Another example of 2D motion is **uniform circular motion**. This does not just refer to objects spinning around in circles, but also to objects following a curved path that is the shape of part of a circle, such as a car going around a bend of constant radius.

- Uniform circular motion specifically refers to following a curved path of constant radius at a steady speed.

In this case, despite the speed remaining constant, the velocity is constantly changing. This is because velocity is a vector quantity, so as the object moves around the circle its direction is changing, and therefore its velocity must be changing. This can be seen in Figure 1.27, where the velocity has changed between points A and B.

DID YOU KNOW?

The terms centre of gravity and centre of mass are often confused. There is a slight technical difference, but this is only apparent if the object is in a non-uniform gravitational field.

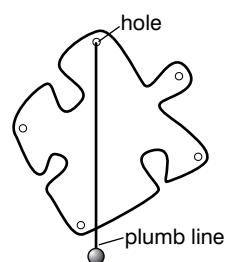


Figure 1.25 Determining the centre of mass of a 2D object.

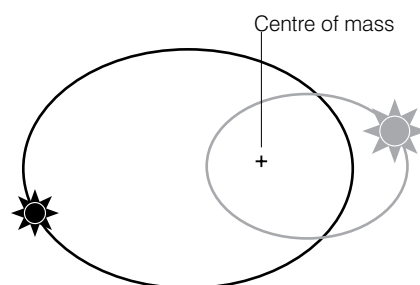


Figure 1.26 Centre of mass of two binary stars.

KEY WORD

uniform circular motion the motion of a body following a curved path of constant radius at constant speed

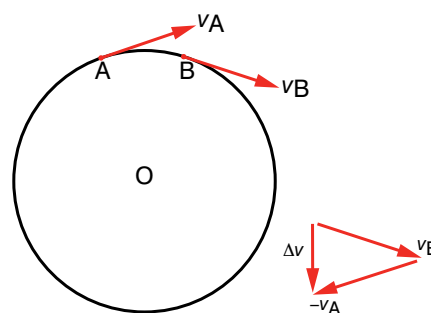


Figure 1.27 The changing velocity for an object moving in a circular path.

KEY WORDS

centripetal force *the net force needed to make a body follow a circular path*

As the velocity is changing, the object must be accelerating (acceleration is defined as the rate of change of velocity). According to Newton's second law, acceleration requires a net force and the direction of the acceleration is the same direction as the net force.

Projectile motion is not an example of uniform circular motion. As the path is a parabola, the radius is constantly changing.

Figure 1.28 shows the direction of the net force required to keep an object following a circular path. This force is referred to as the **centripetal force** and it always acts towards the centre of the circle.

What factors affect the size of the centripetal force?

The mass of the object, its velocity, and the radius of the curved path followed by the object affect the centripetal force.

Activity 1.14: A simple pendulum

Make a simple pendulum using a mass and a piece of string. *Carefully* swing this around your head in a horizontal circle, trying to keep the speed constant.

The force required to make the mass move and follow a circular path will come from your hand. You will be able to feel the force required. Experiment by changing the mass, radius and velocity of the object. How does this affect the force required?

To calculate the centripetal force we use the equation below:

$$F = \frac{mv^2}{r}$$

where F = centripetal force, m = mass of the object, v = velocity of the object, and r = radius of the curved path.

Worked example 1.8

The mass of the Earth is 6.0×10^{24} kg. It travels at a steady speed around the Sun at 30 000 m/s at a radius of 1.5×10^9 m. Find the force required to keep it in orbit.

F (N)	m (kg)	v (m/s)	r (m)
?	6×10^{24}	30 000	1.5×10^9

$$F = \frac{mv^2}{r}$$

$$F = \frac{6.0 \times 10^{24} \text{ kg} \times 30\,000 \text{ m/s}^2}{1.5 \times 10^9 \text{ m}}$$

$$F = 3.6 \times 10^{24} \text{ N}$$

Circular motion examples

There are plenty of examples of circular motion. In each case the centripetal force acts towards the centre of the circular path followed by the object.

Table 1.1 Examples of centripetal forces.

Context	Centripetal force	Direction of the force
The Earth orbiting the Sun	Gravitational attraction	Towards the centre of the circular path: towards the centre of the Sun
An electron orbiting an atom	Electrostatic attraction	Towards the centre of the circular path: towards the centre of the nucleus
A ball on a string being whirled around	Tension in the string	Towards the centre of the circular path: towards the centre of the circle
A car going around a bend	Friction	Towards the centre of the circular path: towards the centre of the bend
A bus going over a hump-backed bridge	A component of weight	Towards the centre of the circular path: towards the centre of the bridge

What if the centripetal forces are not large enough?

There is a maximum centripetal force that can be provided. For example, if the tension gets too high, the string will snap. Likewise, if the friction is not high enough the car may skid or slide.

Imagine the maximum frictional force between the road and the tyres of a certain car is 6500 N. The mass of the car is 1200 kg and the bend has a radius of 85 m. Determine the maximum speed at which the car can take the bend.

$$F = \frac{mv^2}{r}$$

$$v^2 = \frac{Fr}{m}$$

$$v = \sqrt{\frac{Fr}{m}}$$

$$v = \sqrt{\frac{6500 \text{ N} \times 85 \text{ m}}{1200 \text{ kg}}}$$

$$v = 21 \text{ m/s}$$

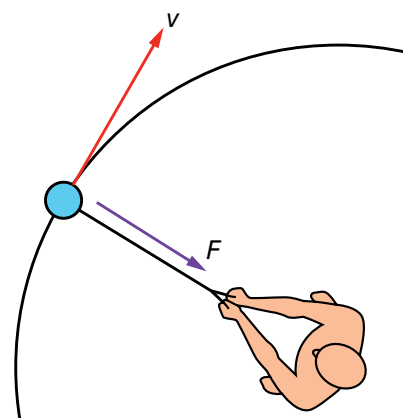


Figure 1.28 Forces acting on a hammer thrower.

Worked example 1.9

A hammer thrower swings a hammer around his head at a steady speed of 2.0 m/s at a radius of 1.2 m. He exerts a constant force of 120 N. Calculate the mass of the hammer.

F (N)	m (kg)	v (m/s)	r (m)
120	?	2.0	1.2

$$F = \frac{mv^2}{r}$$

$$Fr = mv^2$$

$$m = \frac{Fr}{v^2}$$

$$m = \frac{120 \text{ N} \times 1.2 \text{ m}}{2 \text{ m/s}^2}$$

$$m = 36 \text{ kg}$$

Discussion activity

Look carefully at the centripetal force equation. What effect does changing each of the variables have on the force required? For example, what would happen to the required force if the mass doubled? What would happen if the velocity doubled?

If the car travels faster than this, friction will not be large enough to provide the required centripetal force. The car will then follow a path of increased radius, and skid towards the edge of the road.

Summary

In this section you have learnt:

- A projectile is any object moving through the air without an engine or other motive force; examples include tennis balls and rifle bullets.
- Projectile motion and uniform circular motion are examples of 2D motion.
- Gravity causes projectiles to follow a parabolic path; this is because the horizontal velocity remains the same but the vertical velocity increases.
- The angle of a projectile affects its flight path and therefore its range and the time in the air.
- The equations for uniform acceleration can be applied to projectile motion.
- Centre of mass is the point at which all the mass of the object may be considered to be concentrated.
- Uniform circular motion is when an object travels at a steady speed around a circular path.

Review questions

1. This question is about a simple model of the physics of the long jump. Figure 1.29 shows a long-jumper at three different stages, A, B and C, during the jump. The horizontal and vertical components of velocity at each position are shown.

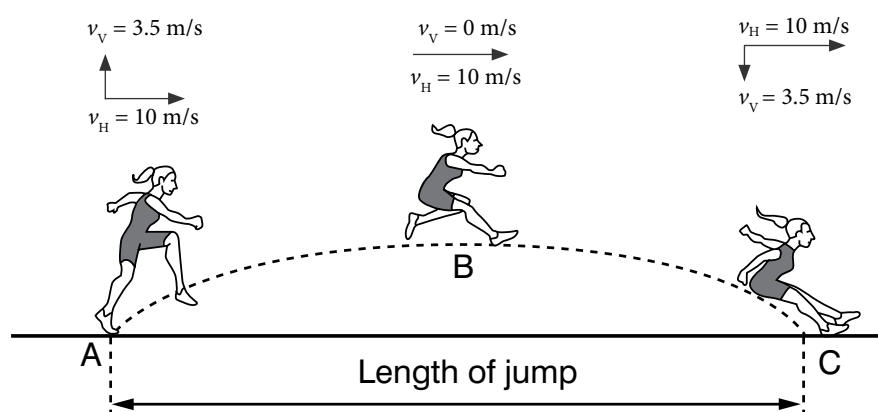


Figure 1.29

- a) i) In the model, the horizontal component of velocity v_H is constant at 10 m/s throughout the jump. State the assumption that has been made in the model.

- ii) Without calculation, explain why the vertical component of velocity v_H changes from 3.5 m/s at A to 0 m/s at B.
 - b)
 - i) By considering only the vertical motion, show that it takes about 0.4 s for the jumper to reach maximum height at B after taking off from A. Take $g = 9.8 \text{ m/s}^2$.
 - ii) Calculate the length in metres of the jump.
 - c) Long-jumpers can use this model to help them to improve their performance. Explain why the length of the jump can be increased by:
 - i) increasing the horizontal component of velocity v_H , keeping v_V the same.
 - ii) increasing the vertical component of velocity v_V , keeping v_H the same.
2. Define the term centre of mass.
 3. Describe in as much detail as you can the motion of a projectile fired at an angle (include descriptions of the vertical and horizontal components of velocity).
 4. A football is kicked at a velocity of 15 m/s at an angle of 25° to the horizontal. Calculate:
 - a) the total flight time
 - b) the maximum height
 - c) the range of the ball.
 5. A car accidentally rolls off a cliff. As it leaves the cliff it has a horizontal velocity of 13 m/s, it hits the ground 60 m from the shoreline. Calculate the height of the cliff.
 6. A car of mass 1200 kg travels at a steady speed of 22 m/s around a bend of radius 50 m. Find the centripetal force required.
 7. Find out what is meant by the centre of stability of a ship. Why is it important that the centre of mass of a ship is below the centre of stability?
 8. A tennis ball machine fires a ball vertically upwards at time $t = 0$ at 19.6 m/s. Assume that air resistance is negligible and take $g = 9.8 \text{ m/s}^2$.
 - a) Calculate the displacement and velocity of the ball at $t = 1.0 \text{ s}$, 2.0 s , 3.0 s and 4.0 s .
 - b) Use your answers to part a) to draw
 - i) a graph of displacement against time for the ball
 - ii) a graph of velocity against time for the ball.

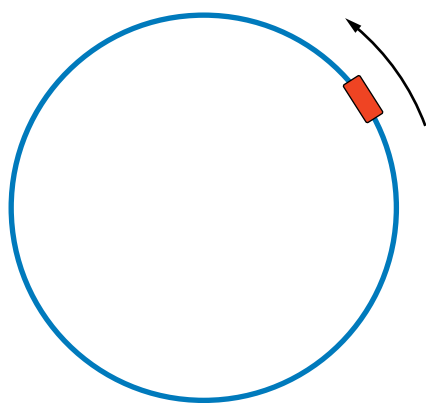


Figure 1.30 Example of circular motion.

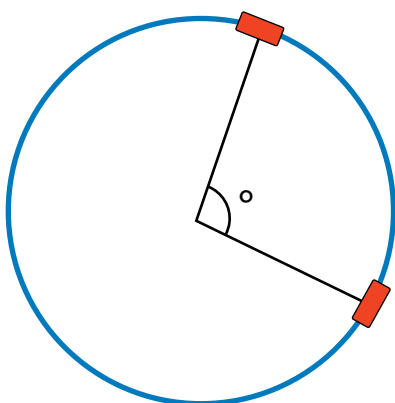


Figure 1.31 Angular displacement.

DID YOU KNOW?

Having 360° in a circle is convenient because it divides easily into a whole a number. It divides by 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, and 360!

Discussion activity

Why are there 360° in a circle? Discuss some possible reasons with a partner.

1.2 Rotational kinematics

By the end of this section you should be able to:

- Define the terms angular and tangential displacement, and angular and tangential velocity.
- Express angles in terms of revolutions, radians and degrees.
- Define the term angular acceleration, and list its key characteristics.
- Identify the SI unit of angular velocity and angular acceleration.
- Explain the relationships between angular displacement, tangential displacement, angular velocity, tangential velocity, angular acceleration and tangential velocity.
- Demonstrate how to use the equations of constant angular acceleration and compare them with equations of constant acceleration.

Rotational motion

Whenever objects travel in curved paths their motion can be considered to be rotational. A simple example might be a ball on a piece of string being swung around in a circular path. A more complex example might include the orbit of the planets around the Sun.

Up until this point, whenever we have used the terms displacement, velocity and acceleration this has always applied to a straight line. We now need to consider the motion of objects following curved paths. Let's take an example of an object travelling in a perfect circle.

As discussed earlier, this object is accelerating (it is changing direction, therefore changing velocity, therefore accelerating). However, what if the object was also getting faster? We need to be able to distinguish between its displacement, velocity and acceleration, and whether it is angular or tangential.

Displacement

In one complete revolution an object will travel a distance equal to $2\pi r$ (where r is the radius of the circle). Its tangential displacement will be zero as it is back where it started. If we consider part of this motion, between two points, we can see that the object has an angular displacement equal to θ . This is just the angle the object has subtended.

When describing angular displacement, there are several different units of angular measurement we could use. Perhaps the simplest would be revolutions. Half a circle would be 0.5 revolutions (or 0.5 rev), two complete loops would be 2 revolutions. However, this does not have much scientific merit. A more common unit is the degree.

One revolution is 360° , half a circle would be 180° , and so on.

For scientific and mathematical calculations the **radian** is used. This has a clear mathematical basis and offers a number of advantages (especially when dealing with high level trigonometry). One radian is the angle subtended at the centre of a circle by an arc that is equal in length to the radius of the circle.

This definition means that there must be 2π radians in a circle. This is because the circumference of a circle is given by $2\pi r$ and so the radius fits around a complete circle 2π times. This could be written as 6.28 rad, but it is usually just written as 2π rad. Half a circle would be π rad (or 3.14 rad).

One radian is equal to $\frac{180}{\pi}$ (or $\frac{360}{2\pi}$) degrees, or about 57.3° .

Table 1.2 Examples of angular measurements

Revolutions	Degrees	Radians
$\frac{1}{8}$	45	$\frac{\pi}{4}$
$\frac{1}{4}$	90	$\frac{\pi}{2}$
$\frac{1}{2}$	180	π
$\frac{3}{4}$	270	$\frac{3\pi}{2}$
1	360	2π
2	720	4π

Velocity

Angular velocity is defined as the rate of change of the angle subtended. This is very similar to the definition of linear velocity (rate of change of displacement). The faster an object rotates, the greater the angle covered per unit of time. This leads us to the relationship:

$$\text{average angular velocity} = \frac{\text{angle covered}}{\text{time taken}}$$

In symbols:

$$\omega = \frac{\theta}{t}$$

ω is the Greek letter omega. As θ is measured in radians and t in seconds, the units of angular velocity are therefore rad/s.

In one complete revolution θ is equal to 2π radians and t becomes T (time for one complete cycle).

So for one complete cycle:

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T}$$

This allows us to work out angular velocities if we know the time taken for one cycle.

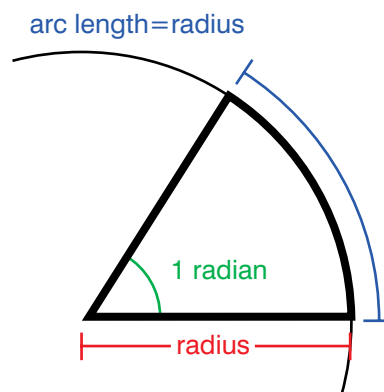


Figure 1.32 The radian.

Activity 1.15: Calculating angular displacement

Thinking about an analogue clock, calculate the angular displacement in revolutions, radians and degrees for the following:

- A second hand after three minutes
- An hour hand after 20 minutes
- The minute hand as it moves from 9.15 to 12.45.

KEY WORDS

radian the angle between two radii of a circle that cut off, on the circumference, an arc equal in length to the radius

angular velocity specifies the angular speed of an object and the axis about which the object is rotating

Activity 1.16: Calculating angular velocity

Other units of angular velocity could be degrees per second or even rpm (revolutions per minute). What would an angular velocity of 3π rad/s be in degrees per second and rpm?

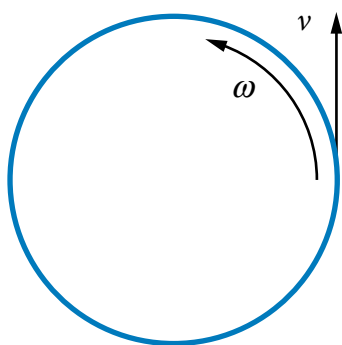


Figure 1.33 Angular and tangential velocity.

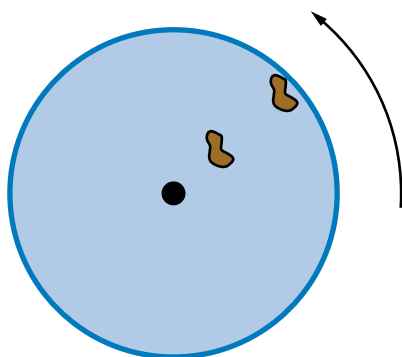


Figure 1.34 Two stones on a spinning disk.

KEY WORDS

acceleration the rate of change of velocity as a function of time

Worked example 1.10

The Earth takes approximately 365 days to orbit the Sun. Calculate its average angular velocity.

θ	T	t
2π	?	?

First we must find the time taken for one orbit in seconds

$$365 \text{ days} = 365 \times 24 \times 60 \times 60 = 3.2 \times 10^7 \text{ s}$$

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T}$$

$$= \frac{2\pi}{3.2} \times 10^7$$

$$= 2.0 \times 10^{-7} \text{ rad/s}$$

As the object follows a circular path it also has a tangential velocity at any given point (see Figure 1.33).

Angular and tangential velocity may be linked using the equation below:

tangential velocity = radius of path $\times$ angular velocity

$$v = r\omega$$

Using the Earth as an example, its tangential velocity at any one point may be calculated:

$$v = r\omega$$

$$v = 1.5 \times 10^{11} \times 2.0 \times 10^{-7}$$

$$v = 30\,000 \text{ m/s}$$

Thinking about this equation further leads us to the conclusion that for any given angular velocity the tangential velocity is proportional to the radius.

Imagine two stones on a spinning disk (see Figure 1.34).

Both stones have the same angular velocity as they both cover the same angle in the same time. However, the stone nearer the edge has a higher speed (as it covers a greater distance in the same time) and so a greater tangential velocity.

Another way to think about uniform circular motion is to define it as any motion where both the speed and the angular velocity are constant.

Acceleration

As discussed in the previous section, any object travelling in a circular path is accelerating. This acceleration is called **centripetal acceleration** and it acts towards the centre of the circle.

Centripetal acceleration is given by the equation:

$$\text{Centripetal acceleration} = \frac{\text{tangential velocity}^2}{\text{radius of curvature}}$$

$$a = \frac{v^2}{r}$$

Since $v = r\omega$ this equation could be written as:

$$a = \frac{r^2 \omega^2}{r}$$

$$a = r\omega^2$$

Again using the Earth as an example, its centripetal acceleration may be found using the following technique:

$$a = r\omega^2$$

$$a = 1.5 \times 10^{11} \times (2.0 \times 10^{-7})^2$$

$$a = 6.0 \times 10^{-3} \text{ m/s}^2$$

Activity 1.17: Calculations using planets

Using the data in the table below, calculate the angular velocity, tangential velocity and centripetal acceleration for each of the planets listed.

Planet	Orbital period (days)	Average distance from Sun(m)
Mercury	88	5.8×10^{10}
Venus	225	1.1×10^{11}
Mars	686	2.3×10^{11}
Jupiter	4330	7.8×10^{11}
Neptune	60 000	4.5×10^{12}

If the mass of an object remains constant, then applying Newton's second law of motion gives us:

$$F = ma.$$

For centripetal acceleration and so centripetal force this equation becomes $F = \frac{mv^2}{r}$.

In the examples we have looked at so far, the object moving in a circular path has been travelling at constant speed. What if the object is also getting faster as it rotates? An example might be a car getting faster as it goes around a bend. In this case its tangential velocity is increasing, and as a result it has a tangential acceleration. The magnitude of this acceleration is given by:

$$\text{tangential acceleration} = \frac{\text{change in tangential velocity}}{\text{time taken}}$$

$$a_T = \frac{\Delta v}{t}$$

In most cases an increase in tangential velocity will lead to an increase of both tangential and centripetal acceleration.

If the radius remains constant, an increase in tangential velocity will also cause an increase in angular velocity. Changes in angular velocity are as common as changes in linear velocity. Just think about a CD spinning up or the wheels of a car as it accelerates.

DID YOU KNOW?

Tangential and angular acceleration are related in the same way as tangential velocity and angular velocity. Instead of $v = r\omega$ we get $a_T = r\alpha$.

Discussion activity

Under what circumstances is it possible to have a constant centripetal acceleration but also be accelerating tangentially?

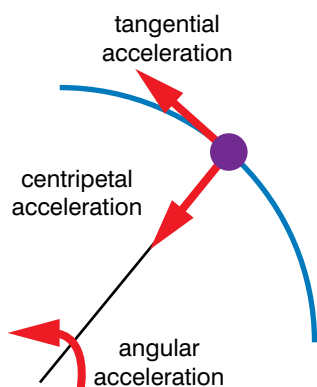


Figure 1.35 Different accelerations in rotational motion.

Angular acceleration is defined in a very similar way to linear acceleration:

$$\text{angular acceleration} = \frac{\text{change in angular velocity}}{\text{time taken}}$$

$$\alpha = \frac{\Delta\omega}{t}$$

α is the Greek letter alpha. As ω is measured in rad/s and t in seconds, the units of angular acceleration are therefore rad/s².

This means that for an object travelling with rotational motion there may be three different types of acceleration.

The equations of constant angular acceleration

You saw on page 5 that there are equations that can be used when there is constant linear acceleration. There is an equivalent set of equations for constant angular acceleration.

Here are the two sets of equations side by side for comparison.

Constant linear acceleration	Constant angular acceleration
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = \frac{1}{2}(u + v)t$	$\theta = \frac{1}{2}(\omega_0 + \omega)t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
$s = vt - \frac{1}{2}at^2$	$\theta = \omega t - \frac{1}{2}\alpha t^2$

ω = final angular velocity

ω_0 = initial angular velocity

α = angular acceleration

t = time

θ = angular distance

Worked example 1.11

Find the final angular velocity when the initial angular velocity is 2 rad/s, the angular acceleration is 3 rad/s² and the time of motion is 4 s.

θ (rad/s)	ω_0 (rad/s)	α (rad/s ²)	t (s)
?	2	3	4

Use $\omega = \omega_0 + \alpha t$

Substitute in given values to get:

$$\omega = 2 + 3 \times 4$$

$$= 14 \text{ rad/s}$$

Worked example 1.12

Find the distance travelled when the final angular velocity is 14 rad/s, the angular acceleration is 3 rad/s² and the time of motion is 4 s.

θ (rad)	ω (rad/s)	t (s)	α (rad/s ²)
?	14	4	3

Use $\theta = \omega t - \frac{1}{2}\alpha t^2$

Substitute in given values to get:

$$\theta = 14 \times 4 - \frac{1}{2} \times 3 \times 4^2 = 56 - 24 = 32 \text{ rad}$$

Worked example 1.13

Find the distance travelled when the initial angular velocity is 2 rad/s, the final angular velocity is 14 rad/s, and the time of motion is 4 s.

θ (rad)	ω_0 (rad/s)	ω (rad/s)	t (s)
?	2	14	4

Use $\theta = \frac{1}{2}(\omega_0 + \omega)t$

Substitute in given values to get:

$$\theta = \frac{1}{2}(2 + 14) \times 4 = \frac{1}{2} \times 16 \times 4 = 32 \text{ rad}$$

Summary

- Angular displacement is the distance an object travels on a circular path and is often measured in radians. Tangential displacement is the distance an object moves in a straight line. Thus, an object that moves round a complete circle will have an angular displacement of 2π radians but a tangential displacement of 0 m (see Figure 1.36).
- Angular velocity is the angular displacement in a given unit of time. It is often measured in radians per second. Tangential velocity is the linear distance moved in a given unit of time and is often measured in metres per second.
- You can express angles in terms of revolutions, radians and degrees. For example, 1 revolution is the same as 2π radians or 360° (see Figure 1.37).
- Angular acceleration is the change in angular velocity per unit time. It is angular acceleration that contributes to the centripetal force which enables objects to move in circular paths. It is often measured in rad/s^2 .
- Tangential displacement and angular displacement are related by the equation:

$$\text{tangential displacement} = \text{radius of path} \times \text{angular displacement}$$
- Tangential velocity and angular velocity are related by the equation

$$\text{tangential velocity} = \text{radius of path} \times \text{angular velocity}$$
- Angular acceleration and angular velocity are related by the equation:

$$\text{angular acceleration} = \frac{\text{angular velocity}}{\text{time}}$$
- The equations of motion with constant angular acceleration are related to the equations of motion with constant linear acceleration as shown in the table on page 28.

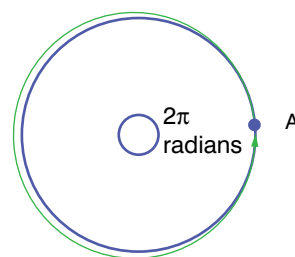


Figure 1.36

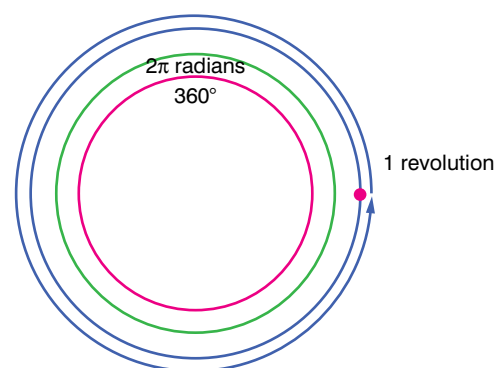


Figure 1.37

Review questions

- Define angular displacement and give its units.
 - Define tangential displacement and give its units.
 - Explain why an object that moves round a complete circle will have an angular displacement of 2π radians but a tangential displacement of 0 m.
- Define angular velocity and give its units.
 - Define tangential velocity and give its units.
 - Find the tangential velocity of an object moving in a path of radius 2 m with an angular velocity of 3 rad/s .

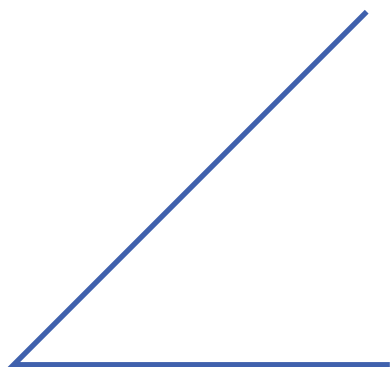


Figure 1.38

3.
 - a) Give the three ways of expressing angles.
 - b) Express the angle shown in Figure 1.38 in three different ways.
4.
 - a) Define angular acceleration and give its units.
 - b) How does angular acceleration enable objects to move in circular paths?
5.
 - a) State the equations of motion with constant angular acceleration.
 - b) Find the final angular velocity when the initial angular velocity is 5 rad/s , the angular acceleration is 2 rad/s^2 and the time of motion is 10 s .
 - c) Find the distance travelled when the final angular velocity is 20 rad/s , the angular acceleration is 2 rad/s^2 and the time of motion is 5 s .
6. A satellite is in orbit 35600 km above the surface of the Earth. Its angular velocity is $7.27 \times 10^{-5} \text{ rad/s}$. What is the velocity of the satellite? (The radius of the Earth is 6400 km .)
7. Astronauts in training are subjected to extreme acceleration forces by the centripetal forces in a giant centrifuge. The radius of the centrifuge is approximately 5 m .
 - a) Calculate the approximate centripetal force on an astronaut of mass 80 kg if the centrifuge rotates once every 2 s .
 - b) Approximately how many times greater than the astronaut's weight is this force?

1.3 Rotational dynamics

By the end of this section you should be able to:

- Define the moment of inertia of a point mass.
- Define rotational kinetic energy of a body.
- Solve simple problems relating to moment of inertia and rotational kinetic energy.
- Define the term torque.
- Identify the SI unit of torque, which is N m , which is not the same as a joule.
- Express torque in terms of moment of inertia and angular acceleration.
- Derive an expression for the work done by the torque.

- Use the formula $W = \tau\theta$ to solve problems related to work done by torque.
- Define the angular momentum of a particle of mass m and write its SI unit.
- State the law of conservation of angular momentum.
- Solve problems using the law of conservation of angular momentum.
- State the first and second conditions of equilibrium.
- Solve problems related to conditions of equilibrium.
- Define the term centre of mass (centre of gravity) of a solid body.
- Determine the centre of gravity using a plumb-line method.
- Define the terms: stable, unstable and neutral equilibrium.

The moment of inertia of a point mass

The **moment of inertia** of a body is a measure of the manner in which the mass of that body is distributed in relation to the axis about which that body is rotating. It is dependent on the:

- mass of the body
- size of the body
- shape of the body
- which axis is being considered.

A spinning flywheel possesses **kinetic energy**, but how much? The expression $KE = \frac{1}{2}mv^2$ applies here, but the difficulty is that different parts of the flywheel are moving at different speeds – the regions further from the axis of rotation are going faster.

With linear motion, the kinetic energy is determined solely by the mass of the body and its speed. With the flywheel, the mass and the angular velocity are important, but there is now a third factor – how that mass is distributed in relation to the axis.

Consider two wheels each of mass M , but one is made in the form of a uniform disc whereas the other consists of a ring of the same radius R fixed to the axle by very light spokes rather like a bicycle wheel (Figure 1.39).

Both are spinning with the same angular velocity, ω . We can easily calculate the kinetic energy stored in the second flywheel because all its mass is at the rim.

You know that for a point mass going steadily in a circle of radius r at a linear speed v , that speed is related to its angular speed by:

$$\omega = \frac{v}{r} \text{ so } v = r\omega$$

KEY WORDS

moment of inertia *a measure of the distribution of the mass of a body in relation to its axis of rotation*

kinetic energy *the work needed to accelerate a body of a given mass from rest to its current velocity*

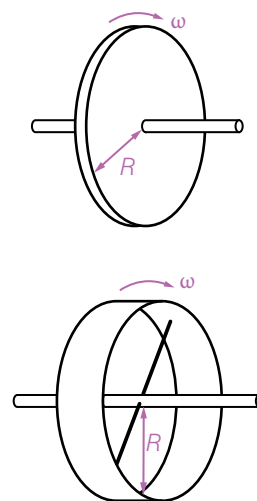


Figure 1.39 Two wheels of the same mass but different construction.

This flywheel has an angular speed ω , and since the whole of its mass is going in a circle of radius R it is all travelling at a linear speed $v = R\omega$. The kinetic energy, $\frac{1}{2}mv^2$, here will total $\frac{1}{2}MR^2\omega^2$.

What can we say about the first flywheel? Its total kinetic energy will be less, because most of its mass is moving at a speed slower than v . How can we go further than that?

A way forward is to think of the wheel as consisting of a large number of separate particles. There is no need to relate them to the individual atoms of the metal – we are just imagining it to be made up of a huge number of very small bits.

One of these bits, of mass m and distance r out from the axis, will have its share of the kinetic energy given by $\frac{1}{2}mv^2$ which can be expressed as $\frac{1}{2}mr^2\omega^2$ (since $v = r\omega$).

Activity 1.18: Rotational inertia

Place two different sized rolls of adding machine tape or other rolled paper on a dowel. Attach heavy clips to the rolls and hold them so that they cannot unwind. Release the rolls at the same time and note which unrolls most rapidly. Which roll has the greatest rotational inertia?

Rotational kinetic energy of a body

The total kinetic energy of the whole flywheel is just the sum of every particle in it. Those particles have different speeds v , but every one has the same angular velocity ω .

Adding all those kinetic energies, and denoting each particle with a subscript 1, 2, 3... and on for ever, we get:

$$\text{Total kinetic energy} = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \frac{1}{2}m_3r_3^2\omega^2 + \dots$$

We can rewrite this as:

$$\frac{1}{2}(m_1r_1^2 + m_2r_2^2 + m_3r_3^2)\omega^2 + \dots$$

The expression in the bracket is the rotational equivalent of mass in the expression of linear kinetic energy ($\frac{1}{2}mv^2$), and we call it the body's moment of inertia, I . Its units will be kg m^2 .

We can simplify it if we replace all those separate values of r^2 by the *average* value of r^2 for *all* the particles in that body. It then becomes the total mass M of the body (which can be measured with an ordinary balance) multiplied by the average value of r^2 for all the particles (which can be calculated by a geometrical exercise for various shapes of body).

The moment of inertia of a body is its rotational inertia. It is not a constant for that body, because it depends on the axis chosen for it to rotate around. It may be defined for a given body about a given axis as the sum of mr^2 for every particle in that body (where m is the mass of the particle and r is its distance from the axis).

For a rotating body, its kinetic energy (KE) = $\frac{1}{2} I \omega^2$

Worked example 1.14

Find the moment of inertia of a point mass of 0.001 kg at a perpendicular distance of 2 m from its axis of rotation.

I (kg m ²)	M (kg)	R (m)
?	0.001	2

Note that the perpendicular distance from a given axis of rotation is specified. This is the value you need for R in the equation.

Use $I = MR^2$

Substitute given values: $I = 0.001 \times 2^2 = 0.004 \text{ kg m}^2$

Worked example 1.15

Find the moment of inertia of a disc of radius 0.25 m and mass 0.5 kg.

I (kg m ²)	M (kg)	R (m)
?	0.5	0.25

Use $I = \frac{1}{2} MR^2$

Substitute in given values: $I = \frac{1}{2} \times 0.5 \times 0.25^2 = 0.015625 \text{ kg m}^2$

Worked example 1.16

Find the moment of inertia of a sphere of mass 0.5 kg and radius 0.15 m.

I (kg m ²)	M (kg)	R (m)
?	0.5	0.15

Use $I = \frac{2}{5} MR^2$

Substitute in given values: $I = \frac{2}{5} \times 0.5 \times 0.15^2 = 0.0045 \text{ kg m}^2$

Worked example 1.17

Find the kinetic energy of a rotating body with moment of inertia 0.004 kg m² and angular velocity of 0.5 rad/s.

KE (J)	I (kg m ²)	ω (rad/s)
?	0.004	0.5

Use $\text{KE} = \frac{1}{2} I\omega^2$

Substitute in given values: $\text{KE} = \frac{1}{2} \times 0.004 \times 0.5^2 = 0.0005 \text{ J}$

Torque

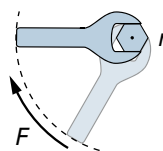
A **torque** is a turning effect. It is the total moment acting on that body about the axis of rotation, and is measured by multiplying the force by its perpendicular distance from the axis.

torque = force $\times$ perpendicular distance

$$\tau = F \times r_{\text{perpendicular}}$$

The linear equivalent of a torque is a force.

The SI unit of torque is N m. This is not the same as a joule!

**Activity 1.19: Comparing linear and rotational kinetic energy**

Make sure that you understand why the expression for kinetic energy of a rotating body is $\frac{1}{2} I\omega^2$ by comparing it with the expression for kinetic energy of a body moving with linear motion.

Some moments of inertia

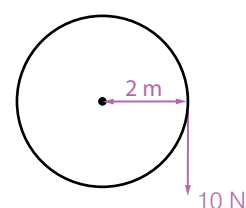
A point mass, M : $I = MR^2$

A disc of mass M and radius R (as with the first flywheel): $I = \frac{1}{2} MR^2$

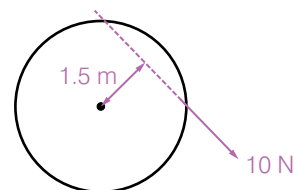
A sphere of mass M and radius R rotating on its axis: $I = \frac{2}{5} MR^2$

KEY WORDS

torque *the tendency of a force to rotate an object about its axis of rotation*



Torque about axis =
 $10 \text{ N} \times 2 \text{ m} = 20 \text{ N m}$



Torque about axis =
 $10 \text{ N} \times 1.5 \text{ m} = 15 \text{ N m}$

Figure 1.40 The value of the torque depends on the distance from the axis.

Activity 1.20: Distance in the torque definition

In a small group, discuss why distance is part of the definition of a torque.

Worked example 1.18

Find the work done when a torque of 5 N m moves through an angle of π radians.

W (J)	τ (N m)	θ (rad)
?	5	π

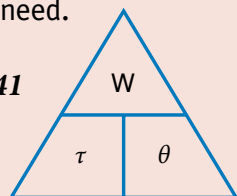
Use $W = \tau \theta$

Substitute given values

$$W = 5\pi = 5\pi \text{ J}$$

Remember that you can use this triangle to rearrange the formula. Cover up the term you are trying to find and the triangle will give you the form you need.

Figure 1.41

**Worked example 1.19**

Find the torque when the work done to move through an angle of π radians is 10 J.

W (J)	τ (N m)	θ (rad)
10	?	π

Use $W = \tau \theta$

Rearrange it so that torque is on the left-hand side.

$$\tau = \frac{W}{\theta}$$

Substitute given values

$$\tau = \frac{10}{\pi} \text{ N m}$$

Torque in terms of moment of inertia and angular acceleration

Since the linear equivalent of a torque, a force, can be found using Newton's second law:

$$\text{force} = \text{mass} \times \text{acceleration}$$

you can see that, using the same law,

$$\text{torque} = \text{moment of inertia} \times \text{angular acceleration}$$

If we give torque the symbol τ , then in symbols this equation is written as:

$$\tau = I\alpha$$

Work done by the torque

In linear terms the work done by a force is Fs , the force (F) multiplied by the distance (s) it moves in that direction. Likewise when a torque turns a body, the work it does is $\tau \theta$, the torque multiplied by the angle it turns through.

Angular momentum of a particle and its SI unit

The linear momentum of a body of mass m moving with a velocity v is defined to be mv . Likewise the **angular momentum** of a body of moment of inertia I rotating at an angular velocity ω is defined to be $I\omega$. Both linear momentum and angular momentum are vector quantities – they have magnitude and direction. Its units are N m s.

The law of conservation of angular momentum

The effect of an unbalanced force on a body is to cause its momentum to change. By Newton's second law of motion the momentum changes at a rate that is proportional to the magnitude of that force, and this leads to $F = ma$.

Similarly, the effect of an unbalanced torque on a body that can rotate is to cause its angular momentum to change, at a rate which is proportional to the magnitude of the torque. This leads to:

$$\tau = I\alpha \text{ (the moment of inertia multiplied by the angular acceleration).}$$

Just as linear momentum is conserved in the absence of a force, so is the angular momentum in the absence of a torque. The conservation of angular momentum says:

- if no resultant torque is acting, the angular momentum of a body cannot change.

This principle is demonstrated readily by a spinning skater, as shown in Figure 1.42. An ice skater holds her arms outstretched to either side and starts to spin round as fast as she can. She then folds her arms and her rate of rotation rises even further. By bringing more of her mass in closer to the axis of rotation, she has reduced her moment of inertia. Since no external torque is acting, $I\omega$ has to stay constant. Here I is made less, so ω has to increase.

Activity 1.21: Experience the conservation of angular momentum

If you have access to a chair that can swivel round freely, you can do this for yourself.

Hold a large mass in each hand and extend your arms fully to either side. Sit in the chair and get your friends to turn the chair round as fast as they can. Tell them to stop turning, and bring the masses close in to your body. Your moment of inertia has fallen so you should spin faster. Your angular momentum is conserved.

Your rotational kinetic energy is $\frac{1}{2} I\omega^2$. This can be written as $\frac{1}{2} (I\omega)\omega$. As you bring your arms in, the $(I\omega)$ term stays constant. You are spinning faster, though, so that extra factor of ω means you have gained in rotational kinetic energy.

Where has it come from? If you have tried this, you may well have felt the answer. As those two masses were pulled into a tighter circle, you had to provide the increasing centripetal force to achieve this. You had to exert a force, and had to move that force through a distance. In other words, you did some work.

Therefore you had to release some of the chemical energy in your food – and that is where the extra kinetic energy came from.

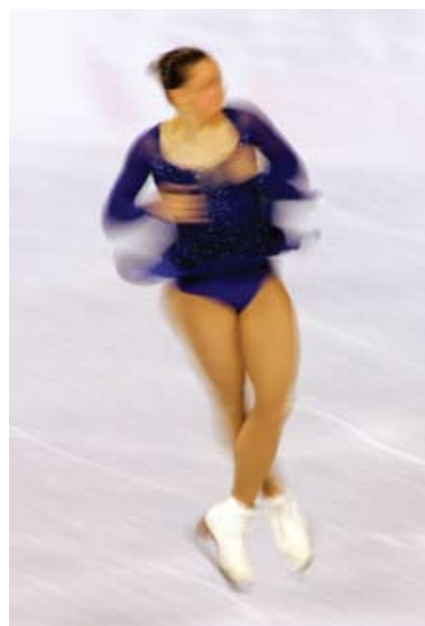


Figure 1.42 Spinning skater.

Activity 1.22: Applications of the principle of conservation of angular momentum

In a small group, make a list of applications of the principle of conservation of angular momentum. To start you off, think about other sports where spinning is required.

Activity 1.23: Comparing linear and rotational motion

In a small group, make a poster to compare linear and rotational motion. You should include all the information you have learnt in this unit, and rotational equivalents of Newton's first and second laws.

Worked example 1.20

No external force acts on a skater. Her moment of inertia is initially 60 kg m^2 . Her angular velocity is 0.2 rad s^{-1} at the beginning of a spin. She brings in her arms and her angular velocity increases so that her moment of inertia decreases to 50 kg m^2 . Find her final angular velocity.

initial angular momentum = final angular momentum

I (kg m^2)	ω (rad/s)
60	0.2

I (kg m^2)	ω (rad/s)
50	?

angular momentum at end of spin = angular momentum at beginning of spin

$$60 \times 0.2 = 50 \times \text{final angular velocity}$$

$$\text{final angular velocity} = \frac{60 \times 0.2}{50}$$

$$0.24 \text{ rad/s}$$

KEY WORDS

equilibrium *a stable situation in which forces cancel one another out*

This table summarises the relationships between linear and angular quantities.

Linear	Angular
$P = mv$	$L = I\omega$
$F = ma$	$\tau = I\alpha$
$F = \frac{\Delta p}{\Delta t}$	$\tau = \frac{\Delta L}{\Delta t}$
$\Delta p = 0$	$\Delta L = 0$
$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$

The conditions of equilibrium

There are two conditions that must be satisfied if a body is to be in equilibrium.

- 1 The forces acting on it must sum to zero.
- 2 The turning effects of the forces must sum to zero.

Worked example 1.21

Explain why these systems are in equilibrium.

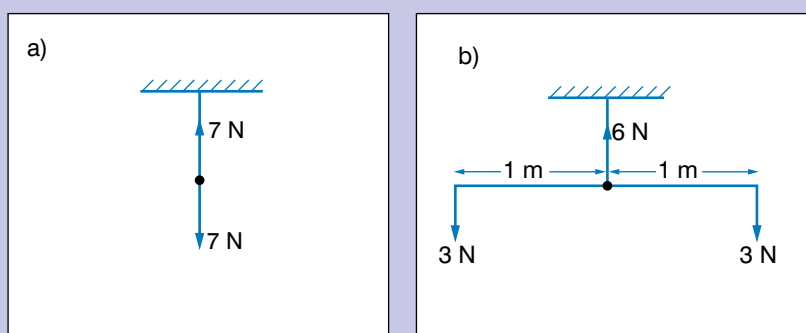


Figure 1.43

In a) there is no turning effect so condition 2 above is satisfied. The forces are 7 N upwards and 7 N downwards so the forces acting sum to zero, satisfying condition 1 above. So a) is in equilibrium.

In b) there would be a turning effect but one is 3 N m anticlockwise and the other is 3 N m clockwise so they sum to zero so condition 2 above is satisfied. The forces are 6 N upwards and a total of 6 N downwards so these sum to zero so condition 1 above is satisfied. So b) is in equilibrium.

The centre of mass (centre of gravity) of a solid body

The **centre of mass** of a solid body is the point at which the body's whole mass can be considered to be concentrated for the purpose of calculations. In a solid body, the position of its centre of mass is fixed in relation to the object (but not necessarily in contact with it). The centre of mass is often called the centre of gravity but this is only true in a system where the gravitational forces are uniform, such as on Earth.

Activity 1.24: Determine the centre of gravity using a plumb-line method

Repeat Activity 1.13 (see page 19) but this time, rather than using a piece of thick card, use an object made from modelling clay.

Stable, unstable and neutral equilibrium

An object is in **stable equilibrium** if, when it is slightly displaced, it returns to its original position.

An object is in **unstable equilibrium** if, when it is slightly displaced, it moves further away from its original position.



Figure 1.45 If this marble is displaced it will run down the ramp and not return to its original position.

An object is in **neutral equilibrium** if, when it is slightly displaced, the system does not necessarily return to its original position but neither does it move further away. For example, if you kick a football along the ground it will roll a little way and then stop at another spot. The kick changes its position but not its stability.



Figure 1.46 This is neutral equilibrium.

DID YOU KNOW?

Engineers try to design a sports car's centre of mass as low as possible to make the car handle better. When high jumpers perform a 'Fosbury Flop', they bend their body in such a way that it is possible for the jumper to clear the bar while his or her centre of mass does not.



Figure 1.44 These spheres will return to their original position when they are displaced.

KEY WORDS

stable equilibrium the tendency of an object, if it is displaced, to return to its original position

unstable equilibrium the tendency of an object, if it is displaced, to keep moving and not to return to its original position

neutral equilibrium the tendency of an object, if it is displaced, to neither return to its original position nor to move farther from it

DID YOU KNOW?

The concept of centre of mass was first introduced by the ancient Greek mathematician, physicist, and engineer Archimedes. He showed that the torque exerted on a lever by weights resting at various points along the lever is the same as what it would be if all of the weights were moved to a single point — their centre of mass. In work on floating bodies he demonstrated that the orientation of a floating object is the one that makes its centre of mass as low as possible.

DID YOU KNOW?

The centre of mass on an aircraft significantly affects the stability of the aircraft. To ensure the aircraft is safe to fly, the centre of mass must fall within specified limits.

Summary

- The moment of inertia of a point mass, M , is $I = MR^2$.
- The moment of inertia for a disc of mass M and radius R is: $I = \frac{1}{2} MR^2$.
- The moment of inertia for a sphere of mass M and radius R rotating on its axis is: $I = \frac{2}{5} MR^2$.
- The rotational kinetic energy of a body is $\frac{1}{2} I \omega^2$.
- You can use the above formulae for simple problems relating to moment of inertia and rotational kinetic energy.
- A torque is a turning effect. It is the total moment acting on that body about the axis of rotation, and is measured by multiplying the force by its perpendicular distance from the axis. $\tau = F \times r_{\text{perp}}$. Its unit is N m, which is not the same as a joule.
- torque = moment of inertia $\times$ angular acceleration
- If we give torque the symbol τ , then in symbols this equation is written as:

$$\tau = I\alpha$$

- When a torque turns a body the work it does = $\tau \theta$, the torque multiplied by the angle it turns. You can use this formula $W = \tau \theta$ to solve problems related to work done by torque.
- The angular momentum of a body of moment of inertia I rotating at an angular velocity ω is defined to be $I\omega$. Its units are N m s.
- The conservation of angular momentum says: if no resultant torque is acting, the angular momentum of a body cannot change.
- You can use this law to solve problems.
- There are two conditions that must be satisfied if a body is to be in equilibrium.
 - 1 The forces acting on it must sum to zero.
 - 2 The turning effects of the forces must sum to zero.
- You can solve problems using the conditions of equilibrium.
- The centre of mass of a solid body is the point at which the body's whole mass can be considered to be concentrated for the purpose of calculations. You can determine the centre of gravity using a plumb-line method.
- An object is in stable equilibrium if, when it is slightly displaced, it returns to its original position.
- An object is in unstable equilibrium if, when it is slightly displaced, it moves further away from its original position.

- An object is in neutral equilibrium if, when it is slightly displaced, the system does not necessarily return to its original position but neither does it move further away.

Review questions

1. Define the moment of inertia of a point mass.
2. Define rotational kinetic energy of a body.
3. a) Find the moment of inertia of a point mass of 0.005 g at a perpendicular distance of 3 m from its axis of rotation.
b) Find the moment of inertia of a sphere of mass 0.3 kg and radius 0.6 m.
c) Find the kinetic energy of a rotating body with moment of inertia 0.003 kg m^2 and angular velocity of 0.6 rad s^{-1} .
4. Define the term torque and identify its SI unit.
5. Express torque in terms of moment of inertia and angular acceleration.
6. Derive an expression for the work done by the torque.
7. a) Find the work done when a torque of 3.5 N m moves through an angle of $\frac{1}{2}\pi$ radians.
b) Find the torque when the work done to move through an angle of $\frac{1}{4}\pi$ radians is 3 J.
8. Define the angular momentum of a particle of mass m and write its SI unit.
9. State the law of conservation of angular momentum.
10. Give examples of uses of the law of conservation of angular momentum.
11. State the first and second conditions of equilibrium.
12. Explain why the system in Figure 1.47 is in equilibrium?

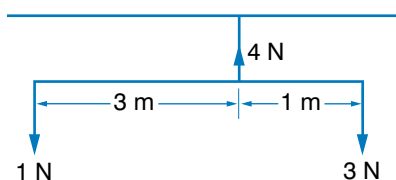


Figure 1.47

13. Define the term centre of mass (centre of gravity) of a solid body.
14. Explain how you can determine the centre of gravity using a plumb-line method.
15. Define the terms stable, unstable and neutral equilibrium.

1.4 Newton's law of universal gravitation

By the end of this section you should be able to:

- State Newton's law of universal gravitation.
- Determine the magnitude of the force of attraction between two masses separated by a distance r .
- Calculate the value of g at any distance above the surface of the Earth.
- State Kepler's laws of planetary motion.
- Use Kepler's laws of planetary motion to determine the period of any planet.
- Differentiate between the orbital and escape velocity of a satellite.
- Determine the period of a satellite around a planet.
- Calculate the orbital and escape velocity of a satellite.
- Describe the period, position and function of a geostationary satellite.

Newton's law of universal gravitation

To simplify matters we shall consider that the planets go round the Sun in circular orbits, which is nearly the case. There is no force opposing their motion, which is why they just keep going.

There has to be a force pulling them towards the Sun, though. Otherwise they would keep moving in a straight line. If the planet has a mass m , is travelling at a speed v and follows an orbit of radius r , the magnitude of the force straight towards the Sun has to be $\frac{mv^2}{r}$.

There is nothing but space between the planet and the Sun. What is producing a force that size? The answer is gravitation. This is a force of attraction that exists between any two lumps of matter, but which becomes noticeable only if at least one of them is of astronomical size.

Based on observations of the path of the Moon, Newton proposed a formula to describe how this force must behave. In doing so, he imagined that the laws of nature that applied to objects on Earth also applied to heavenly bodies. At the time this was a very daring idea.

The law has since been confirmed by incredibly sensitive experiments carried out between a pair of masses on Earth, and is known as Newton's universal law of gravitation.

If two masses M_1 and M_2 are a distance r apart, Newton claimed that the gravitational force F between them was proportional to each of the masses, and decreased as they moved apart by an inverse square relationship – move three times as far apart and the force drops to one ninth, for example.

Putting this together, we get:

$$F = \frac{GM_1M_2}{r^2}$$

G is a constant for all matter everywhere, and is called the gravitational constant. You should be able to see that it will have units of $\text{N m}^2 \text{kg}^{-2}$.

Its value has been measured to be $6.67 \times 10^{-11} \text{ N m}^2 \text{kg}^{-2}$. There is no need to try to remember that, but its small size does illustrate the weakness of the force.

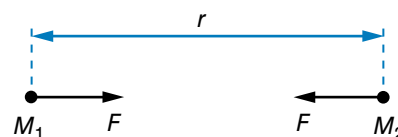


Figure 1.48 A gravitational force attracting two masses.

Worked example 1.22

Find the gravitational force between the Earth and the Moon. The mass of the Moon is $7.35 \times 10^{22} \text{ kg}$, the mass of the Earth is $5.98 \times 10^{24} \text{ kg}$ and the distance between them is approximately $3 \times 10^6 \text{ km}$.

F (N)	G ($\text{N m}^2 \text{kg}^{-2}$)	M_1 (kg)	M_2 (kg)	r (m)
?	6.67×10^{11}	7.35×10^{22}	5.98×10^{24}	3×10^9

Use $F = \frac{GM_1M_2}{r^2}$

Substitute in given values and value for G .

$$F = \frac{6.67 \times 10^{-11} \text{ N m}^2 \text{kg}^{-2} \times 7.35 \times 10^{22} \text{ kg} \times 5.98 \times 10^{24} \text{ kg}}{(3 \times 10^9 \text{ km})^2}$$

$$F = \frac{2.93 \times 10^{37}}{9 \times 10^{18}} = 3.257 \times 10^{18} \text{ N}$$

Finding the value of g at any distance above the surface of the Earth

The value of g varies a little above the surface of the Earth. The law of universal gravitation tells us that the force on a body acted upon by Earth's gravity is given by:

$$F = G \frac{m_1m_2}{r^2} = \left(G \frac{m_1}{r^2} \right) m_2$$

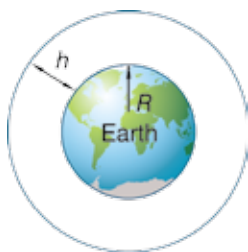


Figure 1.49 To calculate g at a height h above the Earth, use $R + h$ instead of r in the formula.

where r is the distance between the centre of the Earth and the body, and m_1 is the mass of the Earth and m_2 is the mass of the body.

Newton's second law, $F = ma$, where m is mass and a is acceleration, tells us that:

$$F = m_2 g$$

Comparing the two formulae you can see that:

$$g = G \frac{m_1}{r^2}$$

To calculate the value of g at a distance h above the surface of the Earth, you need to substitute the value of the radius of the Earth R , + h , for r in the above formula (see Figure 1.49).

Worked example 1.23

Find the value of g at a distance of 2 km above the surface of the Earth.

g (m/s^2)	G ($\text{N m}^2 \text{ kg}^{-2}$)	m_1 (kg)	r_2 (m)
?	6.67×10^{-11}	5.98×10^{24}	$6.4 \times 10^6 + 2000$

$$\text{Use } g = \frac{G m_1}{r^2}$$

$$g = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.4 \times 10^6 + 2 \times 10^3)^2}$$

$$g = \frac{3.98 \times 10^{14}}{(6\,402\,000)^2}$$

$$g = \frac{3.98 \times 10^{14}}{4.099 \times 10^{13}}$$

$$g = 9.71 \text{ m/s}^2$$

KEY WORDS

Kepler's laws describe the motion of a planet around the Sun

Kepler's laws of planetary motion

Kepler's laws are experimental laws that describe the rotation of satellites about their parent body. They apply to all the satellites around the Earth, but Kepler's data were collected in the late 1500s – before the telescope had been invented – by closely observing the paths of the planets as they went around the Sun.

Kepler began with many sheets of figures, and amazingly succeeded in spotting three unexpected patterns to them all. These are now known as **Kepler's laws**:

- 1 Each planet moves in a path called an ellipse, with the Sun at one focus.

- 2 The line that joins the Sun to the orbiting planet sweeps out equal areas in equal times.
- 3 The square of the time it takes the planet to go round the Sun (that is, the square of its year) is proportional to the cube of its average distance from the Sun.

This law can be written in equation form as

$$T^2 = \left[\frac{4\pi}{GM} \right] a^3$$

where T is the time it takes the planet to go round the Sun, G is the gravitational constant, M is the mass of the planet and a is the mean distance between the planet and the Sun.

We will look at the first two laws. Figure 1.50 illustrates what they are saying. The two shaded areas are equal.

The second law follows from the conservation of energy. As the planet gets closer to the Sun it speeds up, and as it climbs back to a more distant part of its orbit some of that kinetic energy is transferred into its extra gravitational potential energy.

The planets all have orbits round the Sun that are close to being circles. There are some natural satellites of the Sun that are very different, though – the comets. These are only visible to us as they pass quickly through the part of their orbit which is close to the Sun. Most of the time they are remote from the Sun, in the darkness of space and travelling more slowly.

The path of the Moon as it travels round the Earth is very nearly a circle, though there are times in its orbit when it is closer to us and so appears slightly larger in the sky.

The period, position and function of a geostationary satellite

The Earth nowadays has a large number of satellites in orbit round it. Most were made by humans, but the biggest one of them all is natural – the Moon.

Since the path of the Moon as it travels around the Earth is very nearly a circle, we can say that it has constant speed and is subject to a centripetal force

$$F = \frac{mv^2}{r}$$

where m is the mass of the Moon, v is its speed and r is its distance from the Earth.

If Earth has mass M , Newton's law of universal gravitation tells us that

$$F = \frac{GMm}{r^2}$$

$$\text{So } \frac{mv^2}{r} = \frac{GMm}{r^2}$$

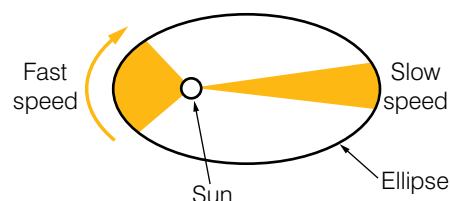


Figure 1.50 Kepler's laws 1 and 2.

Halley's Comet

We know that this has appeared to us in the sky every 75–76 years during recorded history, right back to the year 240 BC. It was last visible in 1986 and is next due in 2061. All the planets go round the Sun in the same direction. Halley's Comet goes the opposite way round, and its orbit is tilted at about 18° to that of the planets. This strongly suggests that it did not form when the planets formed, but passed by the Sun at a later time close enough to find itself captured by it.

$$v^2 = \frac{GM}{r} = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

This equation can be applied to all satellites. This means that the smaller the distance from Earth, the greater the speed of the satellite.

Worked example 1.24

Find the speed of the Moon if the mass of the Earth is 6×10^{24} kg and the distance from the Moon to Earth is 4×10^8 m.

v (m/s)	G (N m ² kg ⁻²)	M (kg)	r (m)
?	6.67×10^{-11}	6×10^{24}	4×10^8

Use $v = \sqrt{\frac{GM}{r}}$

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{4 \times 10^8}}$$

$$v = \sqrt{\frac{4.002 \times 10^{36}}{4 \times 10^8}}$$

$$v = \sqrt{1.0005 \times 10^{28}}$$

$$v = 1.00025 \times 10^{14}$$

KEY WORDS

geostationary satellite

a satellite that orbits the equator with the same angular velocity as the Earth

Geostationary satellites orbits are above the equator and they are going round with the same angular velocity as the Earth. This means that they have the same period as the Earth. They are always in the same spot in the sky and so they are ideal for communications purposes and for satellite navigation systems.

Orbital velocity of a satellite

Other satellites have orbits that are always moving relative to the Earth. Many of these are used to look down on the Earth beneath. Some are military spy satellites, but others observe the Earth – the weather over the whole globe, the temperatures of the oceans and so on. One is even used to provide a platform for a telescope to study the rest of the universe clearly without having to peer through the Earth's atmosphere.

The satellites are all launched from sites as close to the equator as possible, and certainly not from polar regions. The Earth is spinning on its axis: the nearer you are to the equator, the greater the distance you must travel to go round once in a day, and therefore the faster your speed is due to this motion. If you wanted to throw a ball at as great a speed as you could, propelling it forward from a fast-moving car would give you a good start. Satellites launched by humans similarly take advantage of the Earth's motion.

Activity 1.25: Verify Kepler's third law

On page 27, the orbital period of five planets is given. Use this data to verify Kepler's third law.

Activity 1.26: Use of geostationary satellites

Research some uses of geostationary satellites.

A satellite is not always sent into its final orbit in one step. Sometimes it will be launched so it goes into a temporary 'parking orbit'. From there a second carefully controlled rocket can be fired to lift it into its permanent orbit.

All satellites are in orbits that are distant from Earth. There is no mathematical reason why one should not orbit just above the Earth's surface, but the inconvenience of it is not the main reason why nobody does that. The satellites are deliberately positioned so that they are clear of the Earth's atmosphere – this means there is no resistance force to oppose the satellite's horizontal motion, otherwise an engine would be needed to cancel out such a force. Each satellite moves at a particular velocity in its orbit – this is its orbital velocity and is calculated by using the equation $\omega = \frac{2\pi}{T}$ where ω is its angular velocity and T is its period.

Activity 1.27: Calculate the orbital velocity of a satellite

Use $\omega = \frac{2\pi}{T}$ to calculate the orbital velocity of a geostationary satellite.

Escape velocity of a satellite

If a satellite is launched vertically upward at a sufficiently large velocity, it will be able to climb right out of the Earth's potential well and escape completely.

We can calculate this speed. Suppose the satellite has a mass m . At the Earth's surface the potential is $\frac{GM}{R}$ in J kg^{-1} ,

where M and R are the Earth's mass and radius. Therefore the satellite on the Earth's surface will have a potential energy of $\frac{GMm}{R}$ in joules.

When completely clear of the Earth, its potential energy will be zero. Therefore we must raise its potential energy by $\frac{GMm}{R}$ in joules.

If the satellite is launched with a speed of v upwards, it will have an amount of kinetic energy given by $\frac{1}{2}mv^2$. If this kinetic energy is enough to supply what is needed, it can escape.

Thus the minimum escape velocity is given by:

$$\frac{1}{2}mv^2 = \frac{GMm}{R}$$

$$\text{whence } v = \sqrt{\frac{2GM}{R}}$$

The expression for the escape speed can be made even simpler. If we substitute $M = \frac{gR^2}{G}$ in the above, we get:

$$\text{escape velocity } v = \sqrt{2gR}$$

Worked example 1.25

Find the escape velocity for a satellite leaving Earth. The radius of the Earth is $6.4 \times 10^6 \text{ m}$ and g is 9.81 N kg^{-1}

v (m/s)	g (N kg^{-1})	R (m)
?	9.81	6.4×10^6

$$\text{Use } v = \sqrt{2gR}$$

$$v = \sqrt{2 \times 9.81 \times 6.4 \times 10^6}$$

$$v = \sqrt{125\,568\,000}$$

$$v = 11\,205.71 \text{ m/s}$$

$$v = 11 \text{ km/s}$$

Summary

- Newton's law of universal gravitation states that if two masses M_1 and M_2 are a distance r apart, the gravitational force F between them is proportional to each of the masses, and decreases as they move apart by an inverse square relationship:

$$F = \frac{GM_1M_2}{r^2}$$

- G is a constant for all matter everywhere, and is called the gravitational constant. You should be able to see that it will have units of $\text{N m}^2 \text{kg}^{-2}$.
- You can use Newton's law of universal gravitation to determine the magnitude of the force of attraction between two masses separated by a distance r .
- You can calculate the value of g at any distance above the surface of the Earth using:

$$g = G \frac{m_1}{r^2}$$

- To calculate the value of g at a distance h above the surface of the Earth, substitute the value of the radius of the Earth, $R + h$ for r in the above formula.
- Kepler's laws of planetary motion are:
 - Each planet moves in a path called an ellipse, with the Sun at one focus.
 - The line that joins the Sun to the orbiting planet sweeps out equal areas in equal times.
 - The square of the time it takes the planet to go round the Sun (that is, the square of its year) is proportional to the cube of its average distance from the Sun.
- You can use Kepler's laws of planetary motion to determine the period of any planet.
- You can determine the period of a satellite around a planet using $T = \frac{2\pi}{\omega}$.
- Geostationary satellite orbits are above the equator and they are going round with the same angular velocity as the Earth. This means that they have the same period as the Earth. They are always in the same spot in the sky and so they are ideal for communications purposes and for satellite navigation systems.
- The orbital velocity of a satellite is the speed with which it goes round the Earth (or other planet). The escape velocity of a satellite is the speed it needs to escape the potential well of the Earth.

- You can calculate the orbital velocity of a satellite using

$$\omega = \frac{2\pi}{T} \text{ and the escape velocity of a satellite using } v = \sqrt{(2gR)}.$$

Review questions

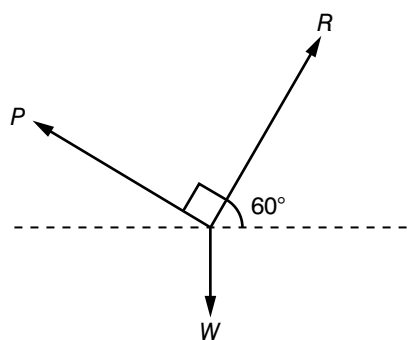
- State Newton's law of universal gravitation.
- Determine the magnitude of the force of attraction between Mercury and the Sun. They are approximately 5.8×10^{10} m apart. The mass of Mercury is about 3.3×10^{23} kg and the mass of the Sun is about 2×10^{30} kg. G is $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.
- Calculate the value of g at 1000 m above the surface of the Earth. The radius of the Earth is 6378.1 km.
- State Kepler's laws of planetary motion.
- Describe the period, position and function of a geostationary satellite.
- Differentiate between orbital and escape velocity of a satellite.

End of unit questions

- Define the term 'projectile'.
 - Give some examples of projectiles.
- Explain why the angle is important when launching projectiles.
 - Find the range of a projectile launched at an angle of 45° with an initial velocity of 25 m/s.
 - Explain why the vertical velocity of a projectile does not change.
- State the centre of mass theorem.
 - Give some practical applications of centre of mass.
- Draw a table to compare the equations of motion with constant angular acceleration with the equations of motion with constant linear acceleration.
- Find the distance travelled when the initial angular velocity is 3 rad/s, the final angular velocity is 25 rad/s, and the time of motion is 10 s.
- Find the moment of inertia of a disc of radius 0.4 m and mass 0.75 kg.
- Copy and complete this diagram so that the system is in equilibrium.



8. This system is in equilibrium. Write an expression linking P , R and W .



9. Determine the magnitude of the force of attraction between the Moon and the Earth. They are approximately 4×10^8 m apart. The mass of the Moon is 2×10^{24} kg and the mass of the Earth is 6×10^{24} kg.
10. Geostationary satellites are placed in orbits of radius 4.2×10^4 km. Use this information to deduce g at that height.
11. A climber of mass 80 kg is on a steep rock face. The force X that the rock exerts on the climber is at an angle of 50° to the vertical. Y , the other force on the climber, keeps him in equilibrium and is provided by a rope at an angle of 40° to the vertical.
- Draw a sketch to show the forces acting on the climber.
 - From your sketch of the forces, sketch a triangle of forces to show equilibrium.
 - Use your triangle of forces to find
 - X , the force the rock face exerts on the climber
 - Y , the force provided by the rope.
12. Which of these answers is correct? Justify your answer.
- If the polar ice caps melt completely as a result of rising global temperatures, then
- the Earth will rotate faster
 - the Earth will rotate slower
 - there will be no change in the angular speed
 - the duration of a day on the Earth will increase.

Contents	
Section	Learning competencies
2.1 Electric charge (page 50)	• State the law of conservation of charge. • Describe and explain the charging processes: charging by rubbing, conduction and induction. • Perform an experiment to charge an electroscope by conduction and by induction. • Describe the distribution of charge on a conductor of variable shape. • Explain how lightning is formed. • Describe the use of a lightning rod. • Describe how equipment works using electrostatics principles. • Describe hazards and uses of electrostatics.
2.2 Electric forces and fields (page 60)	• Define an electric field. • Represent diagrammatically the electric field lines around and between two points. • Distinguish between the electric field inside, outside and between surfaces of a spherical metallic conductor. • State Coulomb's law. • Compare Coulomb's law and Newton's law of universal gravitation. • Calculate the force acting on a charge due to two other charges placed on the same plane (line of action). • Calculate the force between three charges placed in a line. • Calculate the electric field strength at a point due to charges placed in a line and at right angles.
2.3 Electric potential (page 67)	• Define electric potential and its SI unit. • Distinguish between absolute potential and potential difference. • Show that $1 \text{ N/C} = 1 \text{ V/m}$. • Explain equipotential lines and surfaces. • Draw equipotential lines and surfaces in an electric field. • Define the term electric potential energy.
2.4 Capacitors and capacitances (page 70)	• Describe the structure of a simple capacitor. • Define the term capacitance and its SI unit. • Apply the definition of capacitance to solve numerical problems. • Use the circuit symbol to represent a capacitor. • Explain the charging and discharging of a capacitor. • Define the term dielectric and explain what is meant by a dielectric material. • Identify combination of capacitors in series, parallel and series-parallel. • Explain the effect of inserting a dielectric in the gap between the plates of a parallel plate capacitor.

Contents

Section	Learning competencies
	- Derive an expression for the effective capacitance of capacitors connected in series and parallel. - Draw an electric circuit diagram for a simple capacitor, series and parallel connections of two or more capacitors using symbols. - Solve problems on combination of capacitors. - Define parallel plate capacitor. - Describe the factors that affect the capacitance of a parallel plate capacitor. - Calculate the capacitance of a parallel plate capacitor. - Find an expression for the electric potential energy stored in a capacitor. - Calculate the energy stored in a capacitor using one of three possible formulae. - State some uses of capacitors in everyday life.

KEY WORDS

electrostatics *the build up of electric charge on the surface of objects*

charge *an electric charge, positive, negative, or zero, found on the elementary particles of matter*

atoms *the smallest components of an element having the chemical properties of that element*

positive charge *having a deficiency of electrons*

negative charge *having a surplus of electrons*

2.1 Electric charge

By the end of this section you should be able to:

- State the law of conservation of charge.
- Describe and explain the charging processes: charging by rubbing, conduction and induction.
- Perform an experiment to charge an electroscope by conduction and by induction.
- Describe the distribution of charge on a conductor of variable shape.
- Explain how lightning is formed.
- Describe the use of a lightning rod.
- Describe how equipment works using electrostatics principles.
- Describe hazards and uses of electrostatics.

Activity 2.1: Recalling other physical quantities that are conserved

In a small group, spend one minute writing a list of physical quantities that you have met that follow a conservation law. Hint: think back to Unit 1.

Electrostatics in everyday life

Have you ever felt a small electric shock when you have touched a metal door handle, or heard a crackling sound as you pull a woollen article of clothing over your head? Both these arise because of **electrostatics** – electric **charge** is transferred from one material to another. In this section you will learn more about this, its applications and its hazards.

What is electric charge?

All objects are made up from tiny building blocks called atoms. Individual atoms are made up of particles that possess both mass and electric charge. A material such as Perspex is made up from a

huge number of atoms, but although we are well aware of its mass, it does not seem to possess any charge at all. Why is this?

Every bit of mass in every atom adds up so that we feel its mass. In every single atom, however, the two sorts of electric charge cancel one another out: there is the same number of positive charges as negative ones, so we do not notice any overall charge. An 'uncharged' piece of Perspex contains a vast number of charges, but the numbers of the two kinds are equal. Charging it up involves upsetting the balance between positive and negative charges. Adding or removing a few more electrons would do this.

The elementary charge, $e = -1.6 \times 10^{-19} \text{ C}$, is the amount of charge on an electron (negative charge) or on a proton (positive charge). To find the total charge Q , you use $Q = N \times e$, where N is the number of electrons.

Conservation of charge

Electric charge cannot be created or destroyed, only transferred. The number of **positive** charges on an atom of a substance is the same as the number of **negative** charges on the atom. The atom is neutral. The charges from some atoms can be removed from the atom and transferred to another material in the process known as charging but the overall number of positive and negative charges does not change. This is the **law of conservation of charge**.

Like charges will repel each other (for example, two positive charges will repel each other, two negative charges will repel each other). Unlike charges will attract each other (so a positive charge will attract a negative charge and a negative charge will attract a positive charge).

Charging materials by rubbing

It is possible to charge some materials by rubbing them. When you rub a piece of Perspex, for example, some of the charge is transferred from the surface of the Perspex to the material you are using to rub it, and so the overall charge on the Perspex becomes unbalanced.

KEY WORDS

law of conservation of charge *the total electric charge of a system remains constant despite changes inside the system*

DID YOU KNOW?

The fact that some materials (such as amber) become charged after they are rubbed has been known for thousands of years. The Greek word for amber, *ήλεκτρον* (electron), is the source of the word 'electricity'.

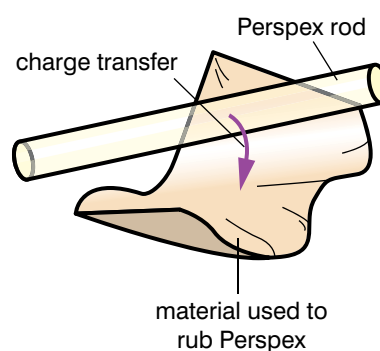


Figure 2.1 Rubbing Perspex transfers charge from the surface of the Perspex to the material you are rubbing it with.

Activity 2.2: Testing how charged bodies attract or repel one another

Take a bar of Perspex and pivot it. One way is to suspend it by a nylon thread (cotton, if the slightest bit damp, may allow the charges to leak off too readily). Charge the suspended rod by rubbing it. Bring a second charged Perspex rod up to the first one, and you will see the first one swing away.

Now charge the bar of Perspex again. Bring a charged rod of Polythene up to the suspended rod, and you will see the suspended rod moving towards it. What does this tell you about the charges on the Perspex and the Polythene?

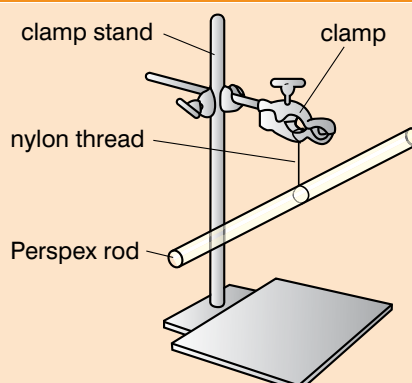


Figure 2.2 Apparatus to test attraction or repulsion between charged bodies.

Activity 2.3: The electrostatic attraction of water

Adjust a tap so a continuous but gentle stream of water is falling from it. Rub a plastic comb on your sleeve, and bring it up to the side of the water column. Describe and try to explain what happens.

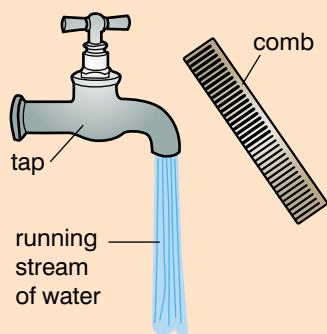


Figure 2.3 The electrostatic attraction of water.

Activity 2.5: Charging an electroscope by conduction

Charge a metal sphere. Bring it into contact with an electroscope. What happens? Draw a diagram to explain your results.

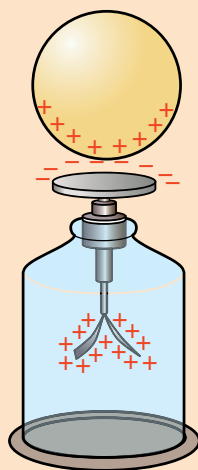


Figure 2.7 Charging an electroscope by conduction.

Activity 2.4: Hanging balloons using electrostatic attraction

Inflate a balloon and then briskly rub one side of it on your hair. Place the surface that you have rubbed towards a wall or door and release it when it appears to be sticking. What can you say about the nature of the surfaces to which the balloon sticks?



Figure 2.4 Rubbing a balloon on hair.

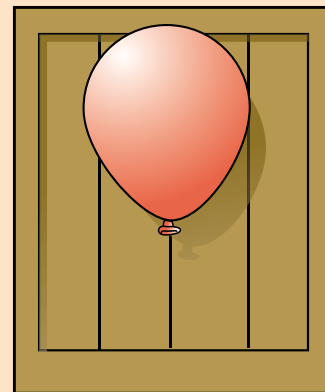


Figure 2.5 A balloon sticking to a surface.

Charging materials by conduction

If a charged object is brought into contact with an uncharged (neutral) object, then the neutral object will become charged by **conduction**. Charge flows from the charged object to the neutral object. (This is what happens when you feel an electric shock when you touch a metal door handle – the door handle has become charged and the charge travels by conduction to you, a neutral object!)

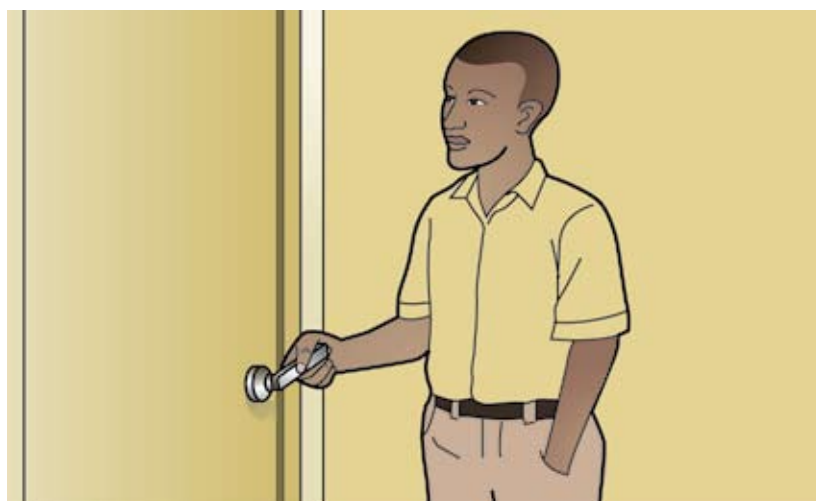


Figure 2.6 Charging a person by conduction.

Charging by induction

Figure 2.8 shows a different way of charging a body known as **induction**.

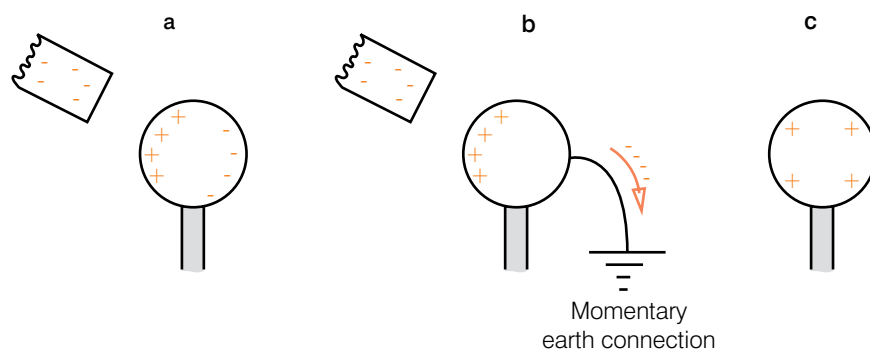


Figure 2.8 Charging by induction.

In Figure 2.8a, a negatively charged rod is brought near to a neutral metal sphere held on an insulating support. The negative charges in the metal sphere move to the side of the sphere furthest from the rod, leaving the side of the sphere nearest to the rod positively charged. In Figure 2.8b, while the rod is still nearby, you touch the sphere for a moment then let go. The negative charges now escape further, through you down to earth. In Figure 2.8c, the negative rod is taken away. The negative charges are unable to return from earth, so the sphere is left positively charged. These charges now distribute themselves evenly.

Activity 2.7: Charging by induction

Tear a sheet of newspaper into small pieces, approximately half a centimetre in diameter. Place the bits under a glass plate that is supported by two books (see Figure 2.10).

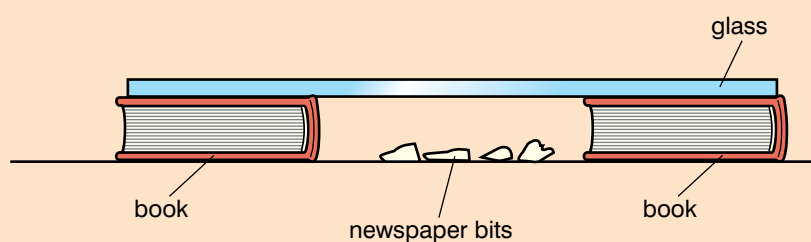


Figure 2.10 Charging by induction.

Rub the glass vigorously with a piece of silk and notice how the paper jumps up to the glass. In a small group, discuss what you see. Answer these questions:

- What attracts the paper to the glass?
- Why does it not stay attached to the glass?

Activity 2.6: Charging an electroscope by induction

From the information given about charging by induction, devise and carry out an experiment to charge an electroscope by induction. Draw diagrams to explain what happens.

DID YOU KNOW?

An electroscope is used to detect the presence and magnitude of electric charge. The British physician William Gilbert invented the first electroscope, which was called the *versorium*, around 1600. This was the first electrical measuring instrument.



Figure 2.9 William Gilbert.

KEY WORDS

conduction the flow of charge from a charged object to an uncharged (neutral) object

induction a redistribution of electrical charge in an object, caused by the influence of nearby charges



Figure 2.12 Dr Robert Van de Graaf and Dr Karl Taylor Compton with a generator.

KEY WORDS

conductor a material that contains movable electric charges

DID YOU KNOW?

The American physicist Robert J. Van de Graaf developed the **Van de Graaf generator** at Princeton University. The first model was demonstrated in October 1929 and used a silk ribbon bought at a local shop as the belt to carry the charge. Accelerating electrons are now used to sterilise food and process materials.

Activity 2.8: Using a Van de Graaf generator to make a fluorescent tube glow

Darken the room. Turn on the generator, and slowly move a fluorescent tube towards the generator. Why does the area between the tube, your hand and the generator glow?

Distribution of charges on the surfaces of conductors

Charges on the surface of a **conductor** are able to flow and spread themselves out evenly. The charges on the surface are all the same type and so they repel each other and move as far apart as possible. The result is that they move outwards until they reach the surface. They can get no further than that unless the insulation of the air breaks down to enable them to escape.

The charges on a sphere will distribute themselves evenly over its surface (see Figure 2.11a). If, however, the body has an irregular curvature (see 2.11b) the charge tends to cluster round the sharply curved parts, with the areas of gentle curvature storing far less of the charge. If the metal body has an actual point on it, the accumulation of charge there will be enormous, and the insulation of the air is more likely to break down there so that all the charge is able to escape from the surface. We refer to this as the action of points.

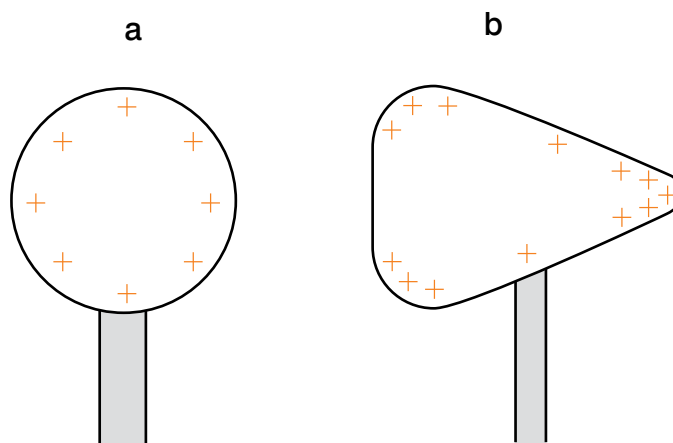


Figure 2.11 Distribution of charges on conductors.

As all the charge sits on the outside, it makes no difference whether the conductor, or metal object, is solid or hollow.

Thunderstorms

Scientists believe that thunderstorms in Ethiopia may sometimes trigger hurricanes as far away as across the Atlantic Ocean.

Thunderclouds form as a result of convection currents (warm air rises and cooler air goes down). Air contains water vapour which we often see as clouds.

As the air cools down, the water vapour in the air condenses. The tiny drops of water that condense experience very rough conditions and, in the process, tend to become charged. The mechanism by which this happens is more than just charging by friction, and we are still not entirely sure of all the details.

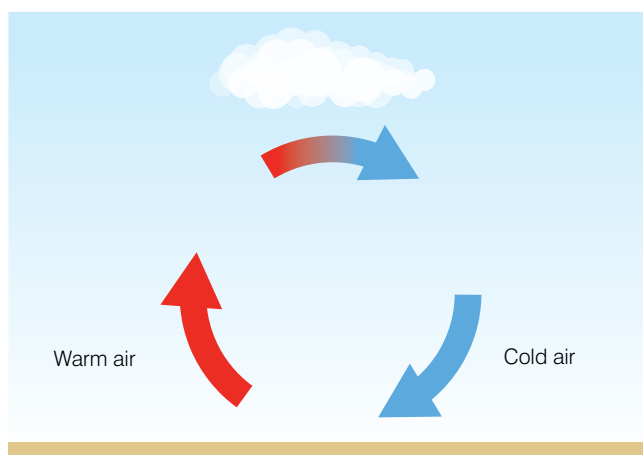


Figure 2.13 A convection current.

How lightning is formed

Charged water drops of one sign (positive or negative) may tend to collect in one part of the cloud. The result is lightning: sparks that jump often from one cloud region to another, but sometimes from the cloud to earth.

When a thundercloud passes overhead, if the base of the cloud is positively charged, then it will attract negative electrons in the earth underneath it (see Figure 2.15). Note that the charge carried by the base of the cloud may be negative or positive but the figure assumes that it is positive.

Should a spark jump to earth it will tend to jump the shortest air gap, such as to trees and tall buildings.

Lightning rods

To help to discharge the cloud safely, a lightning conductor is often fitted to the top of a building. This is a metal bar, pointed at the top, the other end of which is buried firmly in the ground. The idea is that it should get struck before the building and conduct the surge of charge harmlessly to the earth.



Figure 2.16 A lightning conductor will conduct electric charge from lightning safely away from a building to the earth.

DID YOU KNOW?

Fluorescent lamps work because there is a flow of electrical current through a gas (usually argon) which has been charged.

KEY WORDS

Van de Graaf generator
a hollow sphere on which an electrostatic charge is accumulated using a moving belt



Figure 2.14 Lightning.

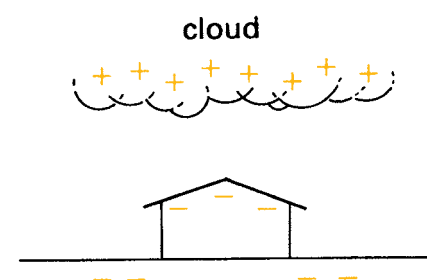


Figure 2.15 How lightning occurs.

Activity 2.9: How might tall buildings be protected from damage by lightning?

In a small group, discuss how tall buildings might be protected from damage by lightning.

Activity 2.10: Why is the inside of a car safe in a storm?

Discuss with a partner why the inside of a car is a safe place to shelter during a storm.

Activity 2.11: Design a safety poster

Use the information above to design a poster to show younger children how to stay safe during a thunderstorm.



Figure 2.17 An electrostatic spray gun in use.

Safety in storms

It is very rare that people are struck by lightning, and certainly you will not be struck while you are inside a car or an aeroplane – the metal shell around you would divert charge away. You are advised not to shelter under trees because they are particularly likely to get struck, and the spark could jump from the lower branches to you.

Applications of electrostatics**Paint sprays**

Electrostatic spray painting is often used to paint car body panels and bike frames. An electrostatic spray painting device sprays charged particles of paint through the air onto a surface. These devices are usually used for covering large surfaces with an even coating of liquid. They can be either automated or hand-held and have interchangeable heads to allow for different spray patterns.

There are three main technologies for charging the fluid (liquid or powders):

- *Direct charging* – an electrode (a charged metal plate) is placed in the paint supply reservoir or in the paint supply conduit.
- *Tribo charging* – this uses the friction of the fluid, which is forced through the barrel of the paint gun. It rubs against the side of the barrel and, in the process, builds up an electrostatic charge.
- *Post-atomisation charging* – the charged fluid comes into contact with an electrostatic field (see page 54) after leaving the spray head. The electrostatic field may be created by electrostatic induction, or by one or more electrodes (electrode ring, mesh or grid).

The charged paint particles repel each other, and so they spread themselves evenly as they leave the spray head. The object being painted is charged in the opposite way (for example, if the paint is positively charged, the object is negatively charged), or grounded. The paint is then attracted to the object, which gives a more even coat than the alternative technique, wet spray painting, and increases the percentage of paint that actually sticks to the object. This method also means that paint covers hard to reach areas. The object is then baked so that the paint sticks properly when the paint powder particles turn into a type of plastic.

Chimney filters

Smoke such as that from a large chimney contains many tiny solid particles, mainly soot. To help to keep the air clean, before reaching the chimney this smoke may be passed between two large upright charged metal plates. In a similar way to how a plastic comb attracts small pieces of paper, the solid specks in the smoke are trapped by sticking to one or other of the plates. From time to time the plates are shaken or scraped to remove the thick layer of dust that has formed on them.

Photocopiers

1. The surface of a cylindrical drum is charged using electrostatics. The drum has a coating of a material that conducts electricity when light shines on it.
2. A bright lamp shines light on the original document, and the white areas of the original document reflect the light onto the surface of the charged drum. The areas of the drum that are exposed to light (those areas that correspond to white areas of the original document) conduct electricity so they discharge. The areas of the drum that are not exposed to light (those areas that correspond to black portions of the original document) remain negatively charged. The result is a stored electrical image on the surface of the drum. (In digital machines, the original document is scanned and digitised and a laser is employed to discharge the drum in a similar fashion.)
3. The toner is positively charged. When it is applied to the drum to produce the image, it is attracted and sticks to the areas that are negatively charged (black areas), just as the balloon in Activity 2.4 stuck to the wall or door.
4. The toner image on the surface of the drum is transferred from the drum onto a piece of paper, which is charged more negatively than the drum.
5. The toner is melted and bonded to the paper by heat and pressure rollers.
6. The drum is wiped clean with a rubber blade and light is used to remove the charge.

When does electrostatic charge cause problems?

Electrostatic charge can be a problem or hazard when there is a sudden discharge. Electrostatic discharge gives a sudden current of electricity between two objects. On page 56, we found out that lightning (which is a form of electrostatic discharge) can be dangerous, especially if you are outside in a thunderstorm. If there is an electrostatic discharge in an area containing liquids or materials which catch fire easily (flammable materials), then the discharge may cause a fire. However, perhaps the main area where electrostatic discharge causes problems is in electronics, where circuits may be damaged by unwanted flows of charge. To protect circuits against this, antistatic devices have been developed. These include antistatic bags, clothing, mats and wrist straps.

Activity 2.12: Design a model chimney filter

In a small group, design a model of chimney filter.

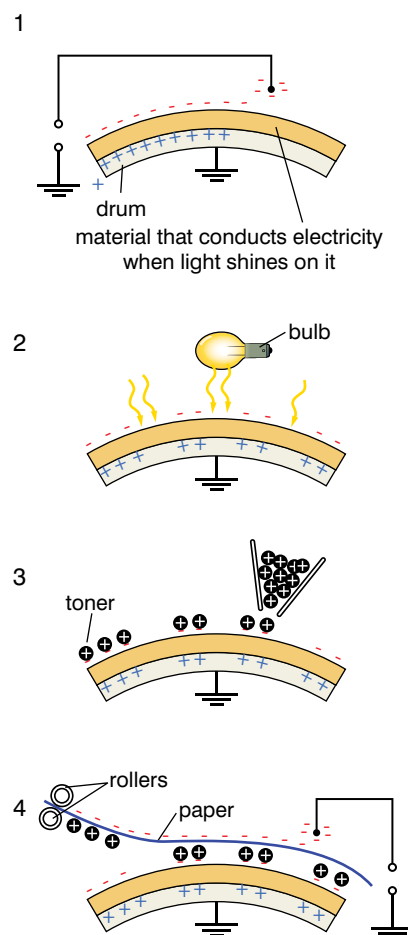


Figure 2.18 How a photocopier works.

Why is static electricity more apparent when the air is dry?

You notice static electricity much more when the air is dry because dry air is a relatively good electrical insulator, so if something is charged the charge tends to stay. When there is moisture in the air, the charged water molecules can remove charge quickly from a charged object.

Activity 2.13: Mind mapping uses and hazards of electrostatics

In a small group, discuss what you have learnt about the uses and hazards of electrostatics. You may like to do some research to find out how electrostatics is used in manufacturing, or more about antistatic devices. Make a summary of your discussion and research using mind maps such as the one in Figure 2.19.

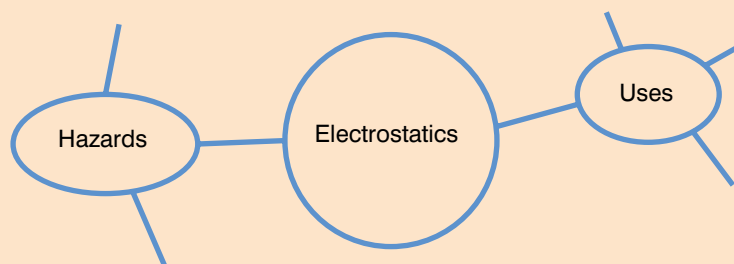


Figure 2.19 Mind map to show uses and hazards of electrostatics. Prepare a presentation for the rest of your class.

Summary

- Charge cannot be created or destroyed, it can only be transferred from one object to another. This is the law of conservation of charge.
- It is possible to charge objects by rubbing, conduction and induction.
- An electroscope can be charged by conduction and induction.
- The distribution of charge on a conductor with a constant surface such as a sphere is uniform (the charges spread out evenly). However, if there is a point on the surface, the charge will form clusters around the point.

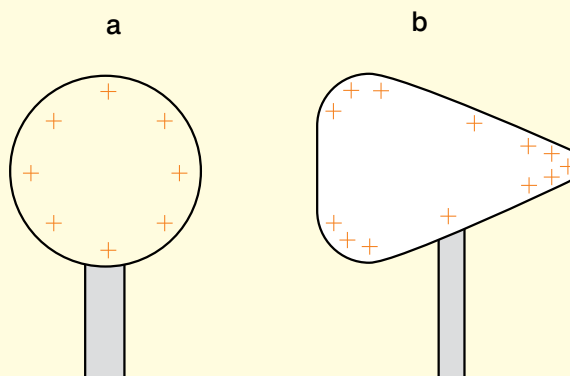


Figure 2.20 Distribution of charge on conductors of various shapes.

- Lightning is formed when charged water drops of one sign (positive or negative) collect in one part of the cloud. There is electrostatic discharge (in the form of sparks) which jump often from one cloud region to another, but sometimes from the cloud to earth.
- A lightning rod may be used to protect tall buildings from lightning. This is a metal rod with a point at the top. It attracts the charge from the lightning and conducts it safely to earth rather than it going through the building and causing damage.
- Electrostatics is used in equipment such as paint sprayers and photocopiers. However, it can be a problem in some situations, such as where there are flammable materials, which may catch fire if there is an electrostatic spark, or in sensitive electronic devices. There are various antistatic devices available, which can be used in different situations.

Review questions

1. Why do charged objects soon lose their charge on a damp day?
2. All atoms that make up matter contain tiny charges, both positive and negative. How is it that most objects we meet in daily life appear to be uncharged?
3. Copy Figure 2.21 and show on it how the charges in the uncharged object will distribute themselves in the presence of the positively charged rod.

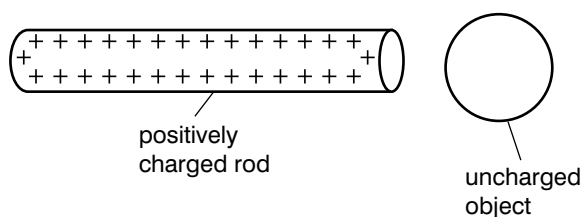


Figure 2.21

4. Why will a plastic comb that has been rubbed attract small pieces of paper?
5.
 - a) Why do plastic objects frequently become covered with dust?
 - b) Why is it not necessarily a good idea to rub such objects with a duster?
6. When car manufacturers apply paint to new cars they usually spray it in very fine droplets that are deliberately charged. Why do they do this?
7.
 - a) An inflated toy balloon can be rubbed on your clothing then stuck on the ceiling. Explain why.
 - b) Suggest why the balloon eventually drops off the ceiling.

8. Why are metal objects designed to store charge (e.g. the dome of a Van de Graaf generator) made from large hollow metal spheres?
9. Explain the purpose of a lightning conductor, and draw a diagram to show how it does its job.
10. a) List some uses of electrostatics
b) List some hazards of electrostatics.
11. List some antistatic devices.

2.2 Electric forces and fields

By the end of this section you should be able to:

- Define an electric field.
- Represent diagrammatically the electric field lines around and between two points.
- Distinguish between the electric field inside, outside and between surfaces of a spherical metallic conductor.
- State Coulomb's law.
- Compare Coulomb's law and Newton's law of universal gravitation.
- Calculate the force acting on a charge due to two other charges placed on the same plane (line of action).
- Calculate the force between three charges placed in a line.
- Calculate the electric field strength at a point due to charges placed in a line and at right angles.

KEY WORDS

electric field *a region where an electric charge will experience a force which is due to the presence of other electric charges*

Also see...

The material in this section is related to Unit 4, Section 4.4.

What is an electric field?

Imagine you are a positively charged particle. Whenever you are anywhere near other charges you will find yourself in a world of pushes or pulls. We say you are in an **electric field**.

An electric field is a region where an electric charge will experience a force that is due to the presence of other electric charges.

Clearly a charge will be surrounded by an electric field, because everywhere around it other charges will either be attracted to it or repelled from it, depending on their signs.

Plotting electric field lines

Electric fields can be plotted by indicating the direction of the force on a positive charge, if one happened to be there. Figure 2.22a shows the electric field around a positive charge, and 2.22b shows that round a negative charge.



Figure 2.22 Electric fields around a positive charge, and a negative charge.

Notice two things. First, a negative charge, such as an electron, would move in the opposite way to the direction shown by the arrows. Second, the field is strongest where the lines are closest together.

We can also plot the field around two charges. In Figure 2.23a you can see the field around a pair of opposite charges, while 2.23b shows the completely different situation when both charges are the same sign.

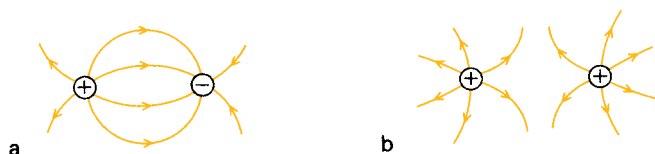


Figure 2.23 Electric fields around a pair of opposite charges, and a pair of charges with the same sign.

Any charge between a pair of charged plates would find itself in a field that was both strong and uniform, as shown in Figure 2.24.

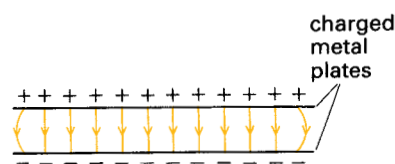


Figure 2.24 The uniform field between a parallel pair of charged plates.

Activity 2.14: Explaining field plots

With a partner, see if you can explain the plots of electric fields shown on this page. Do it by imagining that, in turn, you or your partner are a tiny positively charged particle placed at different points in the field. Think about both the direction of the force you feel and how strong it is.

Also see...

Compare the material on this page to that in Unit 4, Section 4.2.

Electric field inside and outside a spherical metallic conductor

Imagine a spherical, hollow conductor such as the one shown in Figure 2.25.

Activity 2.15: Explaining field patterns

Work with a partner to see whether you can explain the field pattern in Figure 2.25 before reading on!

Inside the conductor there is no charge because the charge has collected on the outer surface. This means that the inside of such a conductor can be used to shield equipment from external electric

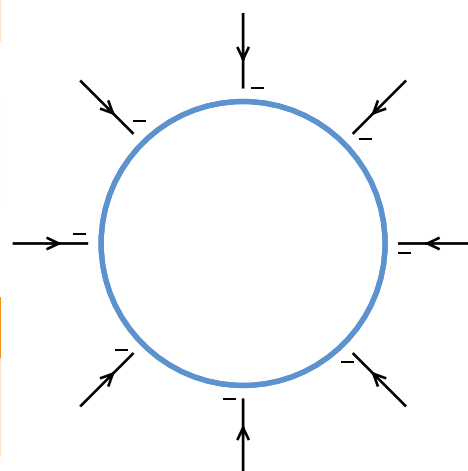


Figure 2.25 Electric field inside and outside a charged hollow spherical metallic conductor.

DID YOU KNOW?

Charles-Augustin de Coulomb, a French physicist, is best known for developing the definition of the **electrostatic force** of attraction and repulsion. This is summarised in Coulomb's law (see page 64). The coulomb is the SI unit of charge and was named after him. During the French Revolution he took part in the determination of weights and measures. He was one of the first members of the French National Institute. As well as his contribution to the understanding of electric fields, Coulomb has contributed to the design of retaining walls!



Figure 2.26 Charles-Augustin de Coulomb, 1736–1806.

KEY WORDS

electrostatic force *the force exerted by stationary objects bearing electric charge on other stationary objects bearing electric charge*

fields. The outer surface of the conductor will have an electric field that is at right angles to (perpendicular to) the surface.

Electric field strength

An electric charge that finds itself in an electric field will experience a force. There are two factors that determine how large a force F this will be:

- The size of the charge Q , measured in **coulombs**.
- The strength (intensity) of the electric field, which corresponds to the closeness of the field lines.

We define the **electric field strength** (electric field intensity) E by:

$$E = \frac{F}{Q}$$

The units of E will be newtons of force per coulomb of charge – that is, N/C; E is a vector – it has direction as well as magnitude.

Worked example 2.1

A charge of 5 C is in an electric field of 3 N C⁻¹. Find the force acting on the charge.

E (N/C)	F (N)	Q (C)
3	?	5

Rearranging $E = \frac{F}{Q}$ gives $F = QE$. Putting in the values, the force acting on the charge will be

$$5 \text{ C} \times 3 \text{ N/C} = 15 \text{ N}.$$

(Notice that the units must work out, as well as the numbers.) If it is a positive charge, the force will act in the direction of the arrow on the field lines. The force on a negative charge will be in the opposite direction.

Worked example 2.2

A negative charge of 8 C is in an electric field of 4 N/C. Find the force acting on the charge. In which direction will it act?

E (N/C)	F (N)	Q (C)
4	?	-8

Rearranging $E = \frac{F}{Q}$ gives $F = QE$. Putting in the values, the force acting on the charge will be

$$8 \text{ C} \times 4 \text{ N/C} = 32 \text{ N}.$$

Since it is a negative charge, the force will act in the opposite direction to the direction of the arrow on the field lines.

Worked example 2.3

Find the electric field strength when a positive charge of 7 C is acted on by a force of 28 N. In which direction will the force act?

E (N/C)	F (N)	Q (C)
?	28	7

Put the given values into $E = \frac{F}{Q}$ so $E = \frac{28}{7}$.

The electric field strength will be 4 N/C.

Since it is a positive charge, the force will act in the direction of the arrow on the field lines.

Worked example 2.4

Find the value, and type, of charge that is acted on by a force of 40 N acting in the opposite direction to the arrow on the field lines in an electric field of strength 10 N C⁻¹.

E (N/C)	F (N)	Q (C)
?	28	7

Rearrange $E = \frac{F}{Q}$ to $Q = \frac{F}{E}$. Put the given values into $Q = \frac{F}{E}$ so $Q = 40/10$. The charge will be 4 C.

Since the force acts in the opposite direction to the the arrow on the field lines, it is a negative charge.

Reminder: rearranging equations

Remember that you can use a triangle like the one in Figure 2.27 to rearrange the equation connecting three quantities.

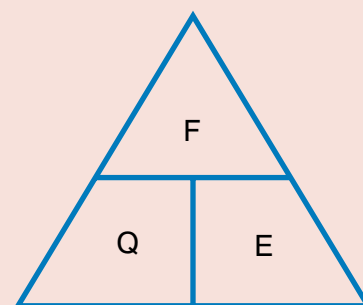


Figure 2.27 To find, for example, Q , cover up Q in the triangle and then you see that $\frac{F}{E}$ is left. This is the formula you need!

KEY WORDS

coulomb unit of electric charge

electric field strength the electrostatic force acting on a small positive test charge placed at that point

Force between charges

We know that two positive electric charges Q_1 and Q_2 a distance r apart will repel each other. This is because each one lies in the field created by the other (see Figure 2.28).

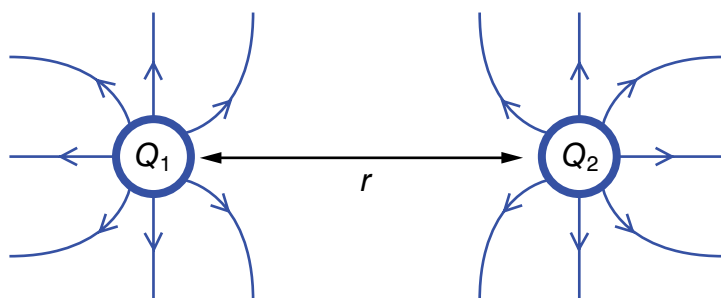


Figure 2.28 Q_1 lies in the field of Q_2 and, since both charges are positive, they will repel each other.

It has been found by carrying out experiments that:

- The force F between the charges increases if the value of either of the charges is increased.
- The force gets less as the distance between the charges gets larger, and does so by an inverse relationship. The force decreases in relation to the inverse of the distance squared, $\frac{1}{r^2}$.

Activity 2.16: Comparing Coulomb's law and Newton's law of universal gravitation

With a partner, recall Newton's law of universal gravitation. (Try to do this without looking back to page 41!) Write the two formulae side by side. Compare them. Make a list of things that are similar and things that are different.

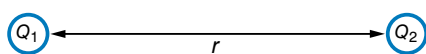


Figure 2.29 Coulomb's law.

This means that if the separation doubles, the force reduces to one quarter; if it increases three times, the force reduces to one ninth, and so on.

Coulomb's law states that the force between two point charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between the two.

We can express this in a formula as $F = \frac{k \times Q_1 Q_2}{r^2}$,

where k is a constant. The newton, the coulomb and the metre are already fixed; that constant has to be obtained by measurement. You might expect the constant to be given a symbol such as 'E', perhaps, but instead it is written as $\frac{1}{4\pi\epsilon_0}$. Do not be put off by this:

it is still a constant, and nothing more complicated than that.

This gives the final expression as:

$$F = \frac{1}{4\pi\epsilon_0} \times \frac{Q_1 Q_2}{r^2}$$

Check that you understand why the units for the constant $\frac{1}{4\pi\epsilon_0}$ will be $\text{N m}^2/\text{C}^2$. The value of this constant depends on the material in which the charges are placed.

Also see...

The material on this page may be compared to that in Unit 1, Section 1.4.

KEY WORDS

Coulomb's law the force between two point charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between the two

Worked example 2.5

The value of $\frac{1}{4\pi\epsilon_0}$ in a vacuum is $9.0 \times 10^9 \text{ N m}^2 \text{C}^{-2}$. What is the force between two charges, 5 C and 7 C, placed 1 metre apart in a vacuum?

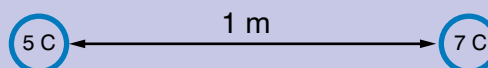


Figure 2.30 A 5 C charge and a 7 C charge placed 1 metre apart in a vacuum.

Use $F = \frac{1}{4\pi\epsilon_0} \times \frac{Q_1 Q_2}{r^2}$

F (N)	Q_1 (C)	Q_2 (C)	r (m)
?	5	7	1

Substitute the given values: $Q_1 = 5 \text{ C}$ and $Q_2 = 7 \text{ C}$, the constant value is $9.0 \times 10^9 \text{ N m}^2/\text{C}^2$ and, since $r = 1$, $r^2 = 1$.

So $F = 5 \times 7 \times (9.0 \times 10^9 \text{ N m}^2/\text{C}^2) = 3.15 \times 10^{11} \text{ N}$

Activity 2.17: Explaining how results of calculations demonstrate the inverse square relationship

Explain to a partner how the results of the two worked examples on this page demonstrate the inverse square relationship suggested in Coulomb's law.

Try to demonstrate the inverse square relationship suggested in Newton's law of universal gravitation using similar examples. (Take the value of $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.)

Worked example 2.6

The value of $\frac{1}{4\pi\epsilon_0}$ in a vacuum is $9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$. What is the force between two charges, 5 C and 7 C, placed 2 metres apart in a vacuum?

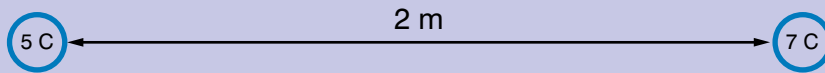


Figure 2.31 A 5 C charge and a 7 C charge placed 2 metres apart in a vacuum.

$$\text{Use } F = \frac{1}{4\pi\epsilon_0} \times \frac{Q_1 Q_2}{r^2}$$

Substitute the given values: $Q_1 = 5 \text{ C}$ and $Q_2 = 7 \text{ C}$, the constant value is $9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$ and, since $r = 2$, $r^2 = 4$.

$$\text{So } F = \frac{[5 \times 7 \times (9.0 \times 10^9 \text{ N m}^2/\text{C}^2)]}{4} = 7.875 \times 10^{10} \text{ N}$$

Coulomb's law when there are more than two charges

Electric force has both size and direction. The two electric forces in Figure 2.32 are the same size but opposite in direction. Charges of the same sign exert repulsive forces on one another, while charges of opposite sign attract, so these two charges would repel each other.



Figure 2.32 Forces of the same size but in opposite directions.

When more than one charge exerts a force on another charge, the total force on that charge is the sum of the individual forces, taking into account both their sizes and direction.

Worked example 2.7

Three charges are arranged in a line with 2.5 cm between them, as shown in Figure 2.33.

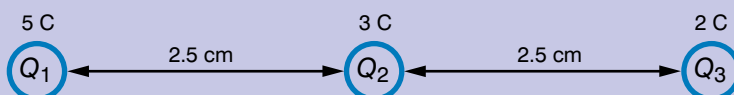


Figure 2.33 Three charges in a line at 2.5 cm apart.

Q_1 is 5 C, Q_2 is 3 C and Q_3 is 2 C, $k = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$.

What is the force exerted on Q_2 by the other two charges?

First draw a good diagram showing the forces acting on the charge. The diagram should also show the directions of the forces.

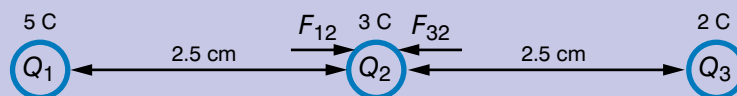


Figure 2.34 Forces acting on a charge with their direction shown.

Consider the forces exerted on Q_2 by the other two:

$$\text{from } Q_1: F_{12} = \frac{kQ_1Q_2}{r^2} = \frac{8.99 \times 10^9 \times 5 \times 3}{(0.025)^2} = 2.1576 \times 10^{14}$$

$$\text{from } Q_3: F_{32} = \frac{kQ_3Q_2}{r^2} = \frac{8.99 \times 10^9 \times 2 \times 3}{(0.025)^2} = 8.6304 \times 10^{13}$$

The total force on Q_2 is therefore $2.1576 \times 10^{14} + 8.6304 \times 10^{13} = 3 \times 10^{14}$ (to 1 significant figure)

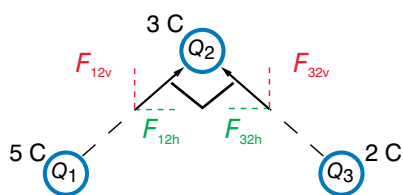


Figure 2.35 Finding the forces between charges.

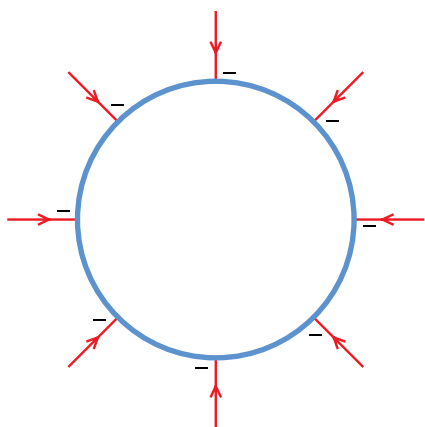


Figure 2.36 An electric field on the inside and outside surfaces.

Note that if Q_1 and Q_3 are at right angles to Q_2 as shown in Figure 2.35, you find the forces between the charges in the same way as in the example above but the lines of action of the forces do not coincide so you have to add the horizontal components of the total force and the vertical components of the total force. Note: F_{12v} is the vertical component of the force between Q_1 and Q_2 .

Summary

- An electric field is a region where an electric charge will experience a force because other charges are present.
- The electric field on the inside and outside surfaces of a spherical metallic conductor is as shown in Figure 2.36.
- Coulomb's law states that the force between two point charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.
- We can express this in a formula as $F = \frac{k \times Q_1Q_2}{r^2}$, where k is a constant.
- Coulomb's law and Newton's law of universal gravitation are both inverse square relationships.
- You can calculate the force between three charges placed in a line.

Review questions

1. Explain, as if to a friend, the meaning of the term 'electric field'.
2. Draw the pattern of the electric field around:
 - a) a positive charge
 - b) a negative charge.

Explain what the arrows mean and explain at which points the strength of the field is greatest.

3. Repeat question 2 for the electric field:
 - a) around a positive and a negative charge
 - b) around two negative charges
 - c) between two oppositely charged flat metal plates.
4. What is the name of the unit we use to measure electric charge? Give its correct abbreviation.
5. How big a force will a charge of +2 C experience if placed in an electric field of strength 18 N/C?
6. In question 5, what can you say about the force if the charge was one of -2 C instead?
7. A charge is placed in an electric field of 48 N/C and it experiences a force of 12 N. How great must the charge be?
8. Two point charges, one of 3.0 C and the other of 2.0 C, are placed 5.0 m apart. Calculate the force which acts between them. (Take $\frac{1}{4\pi\epsilon_0}$ to be $9.0 \times 10^9 \text{ N m}^2 / \text{C}^2$.)
9. The answer to question 8 is an *enormous force* – a coulomb is a huge unit for static charge, and we do not normally encounter point charges of anything like that size. Repeat the calculation but this time take the charges as 3.0 pC (p = pico- = $\times 10^{-12}$) and 2.0 μC .

2.3 Electric potential

By the end of this section you should be able to:

- Define electric potential and its SI unit.
- Distinguish between absolute potential and potential difference.
- Show that $1 \text{ N/C} = 1 \text{ V/m}$.
- Explain equipotential lines and surfaces.
- Draw equipotential lines and surfaces in an electric field.
- Define the term electric potential energy.

DID YOU KNOW?

Alessandro Volta (1745–1827) was an Italian physicist known especially for the development of the first electric cell. Volta made contributions for chemistry: in 1776–77 he discovered methane by collecting the gas from marshes. He devised experiments such as the ignition of methane by an electric spark in a closed vessel.

Volta began to study, around 1791, the ‘animal electricity’ noted by Luigi Galvani when two different metals were connected in series with a frog’s leg and to one another. He replaced the frog’s leg by brine-soaked paper, and detected the flow of electricity.

In 1800, he invented the voltaic pile, an early electric battery, which produced a steady electric current.



Figure 2.37 Alessandro Volta.

Electric potential and its SI unit

Electric potential at a point in space is potential energy divided by charge that is associated with a static (one that does not vary with time) electric field. Its SI unit is the volt, in honour of Alessandro Volta.

You know from earlier in this unit that objects may possess electric charge. An electric field exerts a force on charged objects, accelerating them in the direction of the force, in either the same or the opposite direction of the electric field. If the charged object has a positive charge, the force and acceleration will be in the direction of the field. This force has the same direction as the electric field vector, and its magnitude is given by the size of the charge multiplied with the magnitude of the electric field.

The electric potential created by a point charge q , at a distance r from the charge (relative to the potential at infinity), can be shown to be

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r}$$

where ϵ_0 is the electric constant.

Absolute potential and potential difference

To define absolute potential you need a reference point (for example, infinity). Then you can say that:

electric potential at a point = work done per unit charge in bringing a small object from infinity to a point in an electric field

To define electric potential difference between two points P and Q , you need to assume the absolute electric potential at P is V_P and the absolute electric potential at Q is V_Q . Then the electric potential difference is:

$$V_P - V_Q$$

How to show that two quantities are equivalent

You can show that two quantities are equivalent by using their units.

From the definition of V we can find its units in terms of N, m and C.

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r}$$

We know that the units for the constant term are $\text{N m}^2/\text{C}^2$.

The units for $\frac{q}{r}$ are C/m . So the units for V are $\frac{\text{N m}^2 \text{C}}{\text{C}^2 \text{ m}}$.

This can be simplified, using the laws of indices, to N m/C .

If we divide this by m, to give V/m , we get N/C . So $1 \text{ V/m} = 1 \text{ N/C}$.

Equipotential lines and surfaces

The term ‘equi’ means equal. Equipotential therefore means ‘equal potential’. Equipotential lines are lines showing where the electric potential is the same value. Equipotential surfaces are surfaces that have the same electric potential.

Equipotential lines and surfaces in an electric field

You have already seen a diagram like that in Figure 2.38. The electric field strength along the lines marked in blue is the same at all points along the line. These are therefore lines of equipotential.

Electric potential energy

Potential energy is energy that a body possesses because of its position. If you have a spring and you compress it, you have to do work to make the compression. The work you do is transferred to the spring as potential energy. In a similar way, when a positive charge (say) is moved towards another positive charge, they will repel each other, so work has to be done to push them towards each other. When there is no longer any work being done, the charges will be able to repel each other (the natural state). Electric potential energy is the energy a charge possesses because it is in the region of other charges.

Note that you can convert one form of energy (e.g. electric potential energy) to another form (e.g. kinetic energy, which is energy relating to movement) but energy cannot be created or destroyed. Energy is conserved in a system.

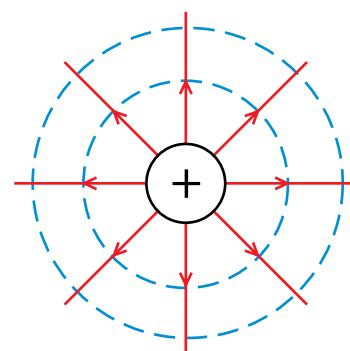


Figure 2.38 Equipotential lines are shown in blue.

Activity 2.18: Drawing equipotentials

Using Figures 2.22b, 2.23a and 2.23b on page 61, draw diagrams showing equipotential lines around

- a negative charge,
- two opposite charges,
- two positive charges.

Summary

- Electric potential at a point in space is potential energy divided by charge that is associated with a static (one that does not vary with time) electric field. Its SI unit is the volt, in honor of Alessandro Volta.
- To define absolute potential you need a reference point (for example, infinity). Then you can say that:
electric potential at a point = work done per unit charge in bringing a small object from infinity to a point in an electric field
- To define electric potential difference between two points P and Q , you need to assume the absolute electric potential at P is V_p and the absolute electric potential at Q is V_q . Then the electric potential difference is:
$$V_p - V_q$$
- You can show that $1 \text{ N/C} = 1 \text{ V/m}$.
- Equipotential lines are lines showing where the electric potential is the same value. Equipotential surfaces are surfaces that have the same electric potential.
- You can draw equipotential lines and surfaces in an electric field.
- Electric potential energy is the energy a charge possesses because it is in the region of other charges.

KEY WORDS

electric potential *potential energy divided by charge that is associated with a static electric field. Its SI unit is the Volt*

Review questions

1. Define electric potential and its SI unit.
2. Distinguish between absolute potential and potential difference.
3. Show that $1 \text{ N/C} = 1 \text{ V/m}$.
4. Explain equipotential lines and surfaces.
5. Draw equipotential lines between two negative charges in an electric field.
6. Define the term electric potential energy.

2.4 Capacitors and capacitances

By the end of this section you should be able to:

- Describe the structure of a simple capacitor.
- Define the term capacitance and its SI unit.
- Apply the definition of capacitance to solve numerical problems.
- Use a circuit symbol to represent a capacitor.
- Explain the charging and discharging of a capacitor.
- Define the term dielectric and explain what is meant by a dielectric material.
- Identify combination of capacitors in series, parallel and series-parallel.
- Explain the effect of inserting a dielectric in the gap between the plates of a parallel plate capacitor.
- Derive an expression for the effective capacitance of capacitors connected in series and parallel.
- Draw an electric circuit diagram for a simple capacitor, series and parallel connections of two or more capacitors using symbols.
- Solve problems on combination of capacitors.
- Define parallel plate capacitor.
- Describe the factors that affect the capacitance of a parallel plate capacitor.
- Calculate the capacitance of a parallel plate capacitor.
- Find an expression for the electric potential energy stored in a capacitor.
- Calculate the energy stored in a capacitor using one of three possible formulae.
- State some uses of capacitors in everyday life.

The structure of a simple capacitor

A capacitor is a small device designed to store more charge at a lower potential. The commonest way of doing this is to use two parallel plates, a tiny distance apart and separated by an insulator (which may be air or may be a dielectric material – see page 73).

Capacitance and its SI unit

If we place some charge on an insulated metal sphere, the sphere's voltage will rise; put some more charge on, and its voltage will rise further.

Presumably a larger sphere will hold more charge before its potential has risen by 1 V. We measure this by using a quantity called capacitance, C . If a charge Q results in a rise of V in the potential, we define the capacitance of the sphere by:

$$C = \frac{Q}{V}$$

This is the charge needed for each volt rise in the sphere potential. The units will be coulombs per volt, C/V, which we call farads, F.

The farad is a very large unit of capacitance – it is rare for an extra coulomb of charge to result in a rise in potential of no more than 1 V. We often use the microfarad (μF , $\times 10^{-6}$ F) or even the picofarad (pF , $\times 10^{-12}$ F).

Worked example 2.8

The metal dome at the top of a Van de Graaf generator will have a capacitance of around 10 pF (1×10^{-11} F). The insulation of air will typically break down when subjected an electric field of around 3000 V/mm. Suppose that the charged dome will produce sparks to earth across an air gap of about 33 mm. That suggests that the potential of the sphere is something like $3000 \text{ V/mm} \times 33 \text{ mm}$, which gives 100 000 V.

C (F)	Q (C)	V (V)
1×10^{-11}	?	1×10^5

The charge held on the dome can be found using

$$C = \frac{Q}{V}$$

Rearranging, $Q = CV$

$$= 1 \times 10^{-11} \text{ F (C/V)} \times 1 \times 10^5 \text{ V}$$

$$= 1 \times 10^{-6} \text{ C (1 } \mu\text{C)}$$



Figure 2.39 A selection of capacitors.

Worked example 2.9

A 150 pF capacitor holds 6.0×10^{-9} C of charge. What will the p.d. be between its terminals?

C (F)	Q (C)	V (V)
150×10^{-12}	6×10^{-9}	?

$$C = \frac{Q}{V}$$

We want to find V . Rearrange to give

$$V = \frac{Q}{C}$$

Substitute values

$$V = \frac{6.0 \times 10^{-9}}{150 \times 10^{-12}} = 40 \text{ V}$$

Larger spheres

A larger sphere will have a bigger capacitance, so you will need to give it a greater amount of charge before its potential rises by 1 V. However, a sphere with a capacitance of 1 F would be vast – its radius would be around 2000 times that of the Earth.

Activity 2.19: Charging and discharging capacitors

Charge a large capacitor for a few minutes by connecting it to a battery, as shown in Figure 2.41.

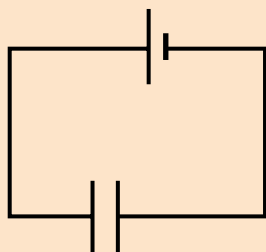


Figure 2.41 Circuit showing capacitor connected to a battery.

Then add a bulb to the circuit, as shown in Figure 2.24.

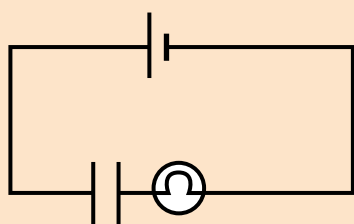


Figure 2.42 Circuit with bulb.

What happens? Try to draw a graph showing light intensity versus time. Discuss what was stored and what was drained before reading on.

Also see...

See page 94 for more on Ohm's law.

The circuit symbol for a capacitor

Figure 2.40 shows the usual circuit symbol for a capacitor.

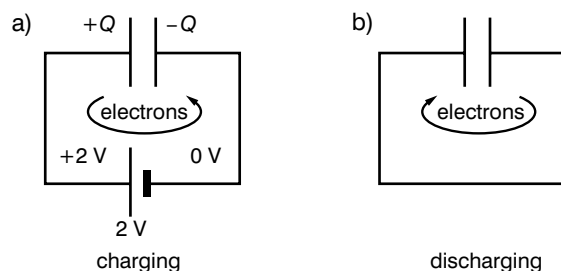


Figure 2.40 Circuit symbol for a capacitor.

In Figure 2.40a, the battery is moving electrons from the plate $+Q$, so it is left with a positive charge. The electrons move round the wires to the $-Q$ plate, which is left with a negative charge. When the potential difference between the plates is the same as that across the battery (2 V), no more charge can flow because the capacitor is balancing the battery.

How much charge is being stored on the capacitor? It might be argued that for example, with $+Q$ on one plate and $-Q$ on the other as shown above, the total amount stored is $2Q$. More plausibly, perhaps, somebody else might say $+Q$ and $-Q$ means a total charge of zero. However, neither is correct. Look at Figure 2.40b which shows the capacitor emptying. What happens is that ' Q ' coulombs of electrons flow off one plate, pass round the circuit in the form of a momentary current and go on to the other plate. The charge stored is Q .

The formula for capacitance is still $C = \frac{Q}{V}$ but the terms now take on a slightly different meaning: Q is the charge held on one of the plates, and V is the potential difference between them.

Charging and discharging a capacitor

When a capacitor discharges through a resistor (or a bulb), the capacitor acts like a battery to drive a current through the resistor, but, unlike a battery, its voltage drops rapidly as its charge drains away. Think of a capacitor charged up to a voltage V_0 which is then emptied through a resistor R . When it is first connected, the discharging current is determined entirely by the resistor (since $I = \frac{V}{R}$ for it) – the higher the value of the resistor, the more slowly the capacitor will empty.

Figure 2.43 indicates how the charge remaining on the capacitor will decrease with time; the vertical axis actually shows the voltage across its plates, but this also provides a measure of the charge left.

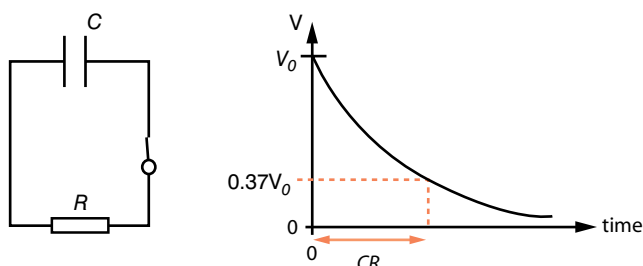


Figure 2.43 Charge decreasing over time.

As the capacitor empties, so the voltage across its plates drops. This decreasing voltage across the resistor R results in a reducing discharge current, so the capacitor empties ever more slowly. The resulting curve is what we call exponential decay.

We can work out how long it takes for the voltage to drop to 0.37 of its starting value (by which time the capacitor has almost two-thirds emptied). We do this by multiplying C by R . This gives what we call the time constant of the circuit.

Working out CR gives a time in seconds. This means that $F\ \Omega$, farads multiplied by ohms, must be seconds. It seems unlikely, but it is certainly a good exercise to try. From the definition of capacitance, be sure you can see why the farad is coulombs per volt, C/V .

The ohm is derived from $R = \frac{V}{I}$, and so is volts per ampere, V/A .

Multiply them together and the unit reduces to C/A . Realise that the coulomb is the ampere second, $A\ s$, and you are there.

When a capacitor is charged through a resistor, we have the opposite to the discharge situation. Here the capacitor charges rapidly at the start, but then continues at an ever-declining rate. It is a kind of 'upside down' exponential curve. When the voltage between the plates of the capacitor equals that of the battery, the charging ceases.

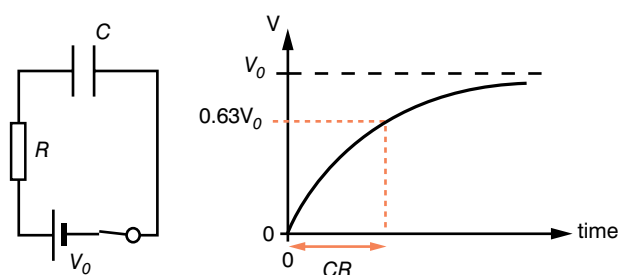


Figure 2.44 Capacitor charging through a resistor.

This time when the time constant has elapsed, the capacitor is almost two-thirds full. Such a circuit can be used as the basis for a timing circuit. When the voltage across the capacitor reaches a certain value, it causes something else to occur – a light to come on, for instance. If a control is provided by which you can adjust the time delay, you will be altering a variable resistor to change the value of R .

Dielectric materials

A **dielectric** is the electrically insulating material between the metallic plates of a capacitor that increases the capacitance of the capacitor (so a greater charge can be stored at a given voltage). The advantage of using a dielectric is that it stops the two charged plates coming into contact with each other.

The dielectric in a capacitor is often a solid material with high **permittivity** (permittivity is a measure of how an electric field affects, and is affected by, a dielectric medium, and relates to a material's ability to transmit (or 'permit') an electric field).

KEY WORDS

dielectric *electrically insulating material*

permittivity *a measure of how an electric field affects, and is affected by, a dielectric medium*

Challenge...

Work in a small group to derive the given formulae for capacitors in series and in parallel.

Combination of capacitors in series, parallel and series–parallel

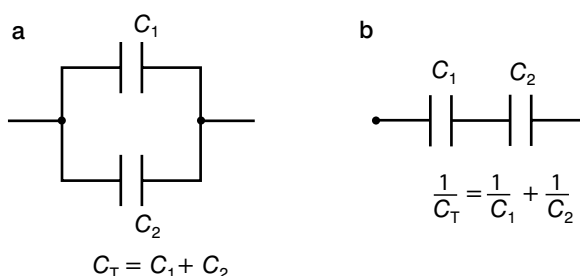
Sometimes you may wish to combine two or more capacitors, to make one of a different value. The formulae are as follows.

Capacitors in parallel (as shown in Figure 2.45a)

$$C_{\text{total}} = C_1 + C_2$$

Capacitors in series (as shown in Figure 2.45b)

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2}$$



$$C_T = C_1 + C_2$$

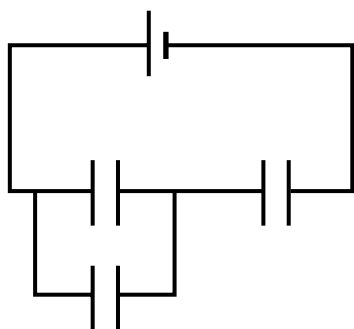


Figure 2.46 Combining parallel and series capacitors.

Activity 2.20: Charging and discharging series and parallel combinations of capacitors

Repeat Activity 2.19, but this time use combinations of capacitors in series and in parallel. Add a resistor to the circuit. Investigate the variation in discharge times. Try to explain your observations.

Figure 2.45 Parallel capacitors and series capacitors.

It is also possible to organise capacitors as shown in Figure 2.46, in which case you work out the capacitance of the parallel combination first and then take that capacitance as C_1 in the formula for series capacitors.

Worked example 2.10

Two capacitors of values $4\ \mu\text{F}$ and $2\ \mu\text{F}$ are placed a) in parallel, b) in series. Draw a diagram to show these circuits. Work out the effective capacitance of each combination.

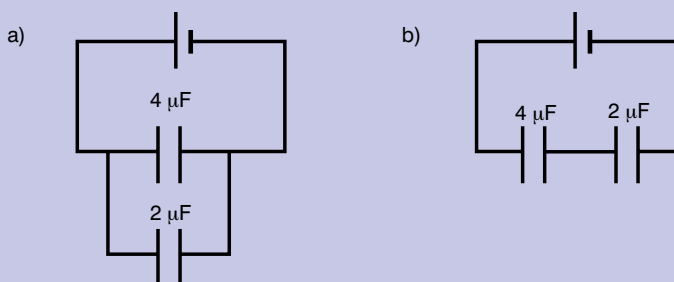


Figure 2.47 Capacitors in parallel use and series use.

a) Capacitors in parallel use $C_{\text{total}} = C_1 + C_2$

$$C_{\text{total}} = 4\ \mu\text{F} + 2\ \mu\text{F} = 6\ \mu\text{F}$$

b) Capacitors in series use

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{\text{total}}} = \frac{1}{4\ \mu\text{F}} + \frac{1}{2\ \mu\text{F}} = \frac{3}{4}\ \mu\text{F}$$

$$\text{So } C_{\text{total}} = \frac{4}{3}\ \mu\text{F}$$

Parallel plate capacitor and the factors that affect its capacitance

A parallel plate capacitor has two plates that are parallel to each other. If a capacitor has two plates each of area A separated by a distance d , then it is possible to calculate what its capacitance C will be by using the relationship:

$$C_0 = \frac{\epsilon_0 A}{d} \quad \text{if the distance } d \text{ is not filled by any dielectric material}$$

If there is a dielectric material,

$$C = \frac{\epsilon A}{d}$$

The symbol ϵ is a constant that varies according to the dielectric which is used to separate the plates.

Looking at the expression, in addition to a suitable dielectric, what are needed for a large capacitance are two plates of large area placed exceedingly close together. One way to achieve this is to use a very long strip of thin metal foil for each of the plates. The two strips are separated by a similar-shaped strip, perhaps of thin waxed paper (which acts as the dielectric). The assembly is then rolled up tightly to fit into a plastic case with two leads, one going to each plate.

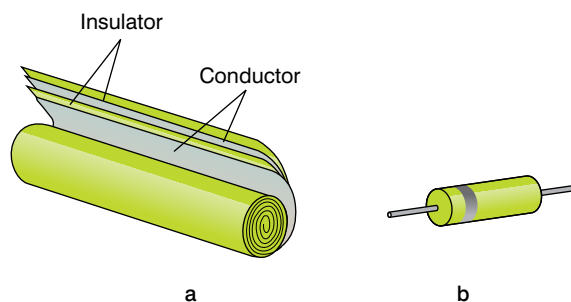


Figure 2.48 Parallel plate capacitor.

Another type of capacitor is the electrolytic capacitor, where the three layers are deposited by an electroplating technique, which helps to make them very thin – so the plates are close together and the whole device is compact. (If you wonder how you can deposit an insulator by electroplating, the answer is you cannot. The dielectric has to be slightly conducting, which means that the charge will soon leak off the capacitor. With some applications in electronics the charge needs to be stored for only a brief interval, in which case that is not important.)

The electric potential energy stored in a capacitor

If a capacitor holds a charge Q at a potential V , then the energy it contains is given by:

$$\text{Energy} = \frac{1}{2} QV$$

An alternative form of the same expression, including $C = \frac{Q}{V}$ as well, is:

$$\text{Energy} = \frac{1}{2} CV^2$$

Activity 2.21

If possible, look for the capacitors in a broken radio. See if you can find the dielectric material.

Worked example 2.11

A $20\ \mu\text{F}$ capacitor is charged to $12\ \text{V}$. How much energy does it contain?

$$\begin{aligned}\text{Energy} &= \frac{1}{2} CV^2 = \\ \frac{1}{2} \times 20 \times 10^{-6} \times 12^2 &= \\ 1.4 \times 10^{-3}\ \text{J} \quad (1.4\ \text{mJ})\end{aligned}$$

This is only a tiny amount of energy. Clearly capacitors are not suitable for storing large amounts of energy.

However, a capacitor can be useful for the flash circuit in a camera. The flash needs a big current just for an instant – not much charge is needed in total, but it must all be supplied quickly. The little battery in the camera cannot supply such a large current, but what it does is to slowly fill up a capacitor; it is the capacitor then that empties to generate the flash for an instant. The capacitor is behaving rather like the tank in a flushing toilet. A small water pipe is used to slowly fill up the cistern, then when it flushes the whole tankful empties in a very short time to give a momentary rapid flow of water.

Why is the energy not given by QV ? After all, we have Q coulombs of charge being stored at a potential of V volts for each coulomb – so why the extra factor of a half?

To see the answer, it is necessary to consider what happened when the charge first went on to the capacitor, a little at a time. The first piece of charge went on to an uncharged capacitor, and so did not have to be ‘lifted’ through any volts. Successive charges had to be ‘lifted’ through an increasing voltage, but it was only the final piece that had to be ‘lifted’ through the full V volts.

Applying those ideas:

the energy stored on the capacitor =

the total charge $\times$ the *average* voltage through which it had to be ‘lifted’ (which is just $\frac{1}{2}V$)

$$= Q \times \frac{1}{2}V = \frac{1}{2}QV.$$

It is a bit like building a brick wall which is to be $2\ \text{m}$ high. The first bricks have to be lifted through no height at all, and it is only the final ones that have to be lifted through the full $2\ \text{m}$. The average brick has to be lifted by $1\ \text{m}$.

Capacitors in everyday life

In direct current circuits (see Unit 3) a capacitor provides a break in the circuit. Apart from a momentary charging current when you switch on, nothing will happen.

In alternating current circuits (see Unit 3) capacitors do useful things, however.

Look at Figure 2.49, which shows a capacitor in a circuit that is being powered by a $50\ \text{Hz}$ alternating voltage supply. This means that the voltage rises to a maximum, dies away, builds up to a maximum in the other direction, then dies away again – and repeats this cycle 50 times every second.

The capacitor is continually charging up, emptying, charging up the other way round then emptying again – and does this 50 times every second. As far as the power supply and the rest of the circuit are concerned, it is just as if there was a complete circuit.

To sum up, therefore, capacitors block direct currents but appear to allow alternating currents to pass. That is one important use in electronic circuits.

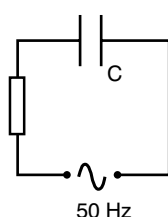


Figure 2.49 Capacitor powered by $50\ \text{Hz}$ alternating voltage supply.

In a public address system, for example, the signal is an alternating current. This has to be passed on from one amplifier stage to the next (see Figure 2.50).

The output of the first stage is at a different voltage to the input of the next stage. Just joining them with a wire would have caused both stages to stop working. The capacitor provides the solution.

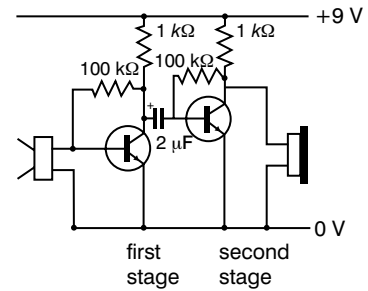


Figure 2.50 Alternating current in a public address system.

Summary

- A capacitor is a small device designed to store more charge at a lower potential. The commonest way of doing this is to use two parallel plates, a tiny distance apart and separated by an insulator.
- If we place some charge on an insulated metal sphere, the sphere's voltage will rise; put some more charge on, and its voltage will rise further.
- Presumably a larger sphere will hold more charge before its potential has risen by 1 V. We measure this by using a quantity called capacitance, C . If a charge Q results in a rise of V in the potential, we define the capacitance of the sphere by:

$$C = \frac{Q}{V}$$

- This is the charge needed for each volt rise in the sphere's potential. The units will be coulombs per volt, C/V, which we call farads, F.
- You can apply the definition of capacitance to solve numerical problems.
- The circuit symbol to represent a capacitor is as shown in Figure 2.51.
- You can use this symbol to draw an electric circuit diagram for a simple capacitor and series and parallel connections of two or more capacitors.
- When a capacitor discharges through a resistor (or a bulb), the capacitor acts like a battery to drive a current through the resistor, but unlike a battery its voltage drops rapidly as its charge drains away.
- When a capacitor is charged through a resistor we have the opposite to the discharge situation. Here the capacitor charges rapidly at the start, but then continues at an ever-declining rate.

It is a kind of 'upside down' exponential curve. When the voltage between the plates of the capacitor equals that of the battery, the charging ceases.

- A dielectric is the electrically insulating material between the metallic plates of a capacitor and increases the capacitance of the capacitor (so a greater charge can be stored at a given voltage). The advantage of using a dielectric is that it stops the two charged plates coming into contact with each other. The dielectric in a capacitor is often a solid material with high permittivity (permittivity is a measure of how an electric field affects, and is affected by, a dielectric medium, and relates to a material's ability to transmit (or 'permit') an electric field).
- Sometimes you may wish to combine two or more capacitors, to make one of a different value. The formulae are as follows:

Capacitors in parallel (as shown in Figure 2.51a)

$$C_{\text{total}} = C_1 + C_2$$

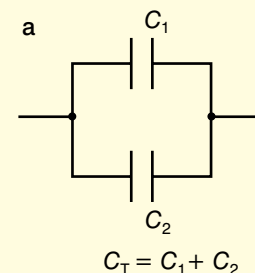


Figure 2.51a

- Capacitors in series (as shown in Figure 2.51b)

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

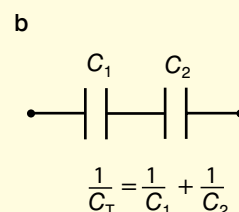


Figure 2.51b

Summary

- It is also possible to organise capacitors as shown below, in which case you work out the capacitance of the parallel combination first and then take that capacitance as C_1 in the formula for series capacitors.

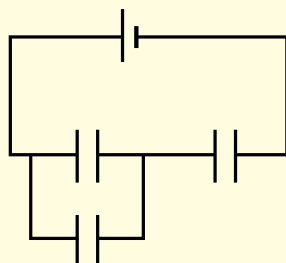


Figure 2.52

- You can use the above expressions to solve problems on combination of capacitors.
- A parallel plate capacitor has two plates that are parallel to each other. If a capacitor has two plates each of area A separated by a distance d , then it is possible to calculate what its capacitance C will be by using the relationship:

$$C = \frac{\epsilon A}{d}$$

The symbol ϵ is a constant that varies according to the dielectric which is used to separate the plates.

- From the expression, the factors that affect the capacitance of a parallel plate capacitor are the dielectric material, the area of the plates, and the distance between the plates.
- You can calculate the capacitance of a parallel plate capacitor using the above formula.
- The electric potential energy stored in a capacitor can be found using

$$\text{Energy} = \frac{1}{2}QV$$
or energy = $\frac{1}{2}CV^2$
- You can calculate the energy stored in a capacitor using one of these formulae.
- Some uses of capacitors in everyday life include in a camera flash, to provide a break in a direct current circuit, to allow alternating currents to pass, to link two stages of an amplifier, and as the basis for a timing circuit.

Review questions

- Describe the structure of a simple capacitor.
- Define the term capacitance and its SI unit.
- A $2.0 \mu\text{F}$ capacitor is charged from a 6.0 V battery.
 - How much charge will it hold?
 - How much energy is stored on it?
 - Where has that energy come from?
 - If the capacitor is then discharged, where does the energy go to?
- Take a $2.0 \mu\text{F}$ and a $5.0 \mu\text{F}$ capacitor and connect them in parallel. Draw the circuit diagram. What is the capacitance of the combination?
 - Repeat part a), but this time connect capacitors in series.
- Explain the charging and discharging of a capacitor.

6. Define the term dielectric and explain what is meant by a dielectric material.
7. Explain the effect of inserting a dielectric in the gap between the plates of a parallel plate capacitor.
8. Define a parallel plate capacitor and describe the factors that affect its capacitance.
9. You want to make a $5.0 \mu\text{F}$ capacitor. For the dielectric you use waxed paper of thickness 0.4 mm and $\epsilon = 7.1 \times 10^{-12} \text{ F/m}$. What area plates will be required?
10. State an expression for the electric potential energy stored in a capacitor.
11. A photographic flash unit has a $100 \mu\text{F}$ capacitor which is charged up by a small 22.5 V battery. How much energy may be discharged into the flash bulb?

End of unit questions

1. Describe how you can test how charged bodies attract or repel one another.
2. Why is the inside of a car a safe place to shelter in a storm?
3.
 - a) Find the force acting on a charge of 4 C in an electric field of 5 N/C .
 - b) State Coulomb's law.
 - c) State the similarities between Coulomb's law and Newton's law of universal gravitation.
4. What is an equipotential?
5.
 - a) What is a capacitor?
 - b) State some uses of capacitors in everyday life.
 - c) Two capacitors of $5 \mu\text{F}$ and $10 \mu\text{F}$ could be arranged either in series or in parallel. Which arrangement gives the highest capacitance? Explain your answer.
 - d) If two capacitors, A and B, have the same area and dielectric constant, but the distance between the plates of A is twice the distance between the plates of B, which has the higher capacitance? Explain your answer.
6. An electron travels at a horizontal velocity of $5 \times 10^6 \text{ m/s}$ in a vacuum between two charged plates 100 mm long and 20 mm apart, to which a p.d. of 30 V is applied. The charge on an electron is $1.5 \times 10^{-19} \text{ C}$ and the mass of an electron is $9.1 \times 10^{-31} \text{ kg}$.
 - a) What size force will the electron experience while it passes through the plates?
 - b) What acceleration will this give to the electron?
 - c) If the positive plate is the lower one, in which direction will the electron accelerate?
 - d) How long will it take for the electron to pass through the plates?

Contents

Section	Learning competencies
3.1 Electric current (page 81)	- Define electric current and its SI unit. - Explain the flow of electric charges in a metallic conductor. - Calculate the number of electrons that pass a point in a given length of time when the current in the wire is known.
3.2 Ohm's law and electrical resistance (page 92)	- Describe factors affecting the resistance of a conductor. - Write the relationship between resistance R, resistivity ρ, length l and cross-sectional area A of a conductor. - Calculate the resistance of a conductor using the formula $R = \rho l/A$. - Find the relationship between resistivity and conductivity. - Construct and draw an electric circuit consisting of source, connecting wires, resistors, switch and bulb using their symbols. - Explain why an ammeter should be connected in series with a resistor in a circuit. - Explain why a voltmeter should be connected in parallel across a resistor in a circuit. - Do experiments using an ammeter and a voltmeter to investigate the relationship between current and p.d. for metallic conductors at constant temperature.
3.3 Combinations of resistors (page 101)	- Identify combinations of resistors in series, parallel and series-parallel connection. - Derive an expression for the effective resistance of resistors connected in series. - Derive an expression for the effective resistance of resistors connected in parallel. - Calculate the effective resistance of resistors connected in series. - Calculate the effective resistance of resistors connected in parallel. - Calculate the current through each resistor in simple series, parallel and series-parallel combinations. - Calculate the voltage drop across each resistor in simple series, parallel and series-parallel connections.
3.4 E.m.f. and internal resistance of a cell (page 108)	- Define the electromotive force (e.m.f.) of a cell. - Distinguish between e.m.f. and terminal potential difference (p.d.) of a cell. - Write the relationship between e.m.f., p.d., current and internal resistance in a circuit. - Use the equation $V = E - Ir$ to solve problems in a circuit. - Identify cell combinations in series and parallel. - Compare the e.m.f. of combinations of cells in series and parallel.

Contents

Section	Learning competencies
3.5 Electric energy and power (page 112)	- Define electrical energy and power in an electrical circuit. - Find the relationship between KWh and joule. - Use $P = VI = V^2/R = I^2R$ to solve problems in electric circuits. - Use $W = VIt = I^2Rt = V^2t/R$ to calculate electric energy dissipated in an electric circuit. - Calculate cost of electrical energy expressed in KWh.
3.6 Electric installation and safety rules (page 115)	- Understand the dangers of mains electricity. - Have some awareness of safety features incorporated in mains electrical installations. - Understand the nature of the generation and supply of electricity in Ethiopia. - Consider employment prospects in Ethiopia's electricity industry.

3.1 Electric current

By the end of this section you should be able to:

- Define electric current and its SI unit.
- Explain the flow of electric charges in a metallic conductor.
- Calculate the number of electrons that pass a point in a given length of time when the current in the wire is known.

KEY WORDS

ampere *SI unit of electric current*

Electric current and its SI unit

An electric current is a flow of charge. Comparing it with water, a small current is like a trickle passing through a pipe; a really large current is like a river in flood.

The rate of flow of electric charge – that is, the electric current – is measured in amperes (A). The **ampere** is one of the fundamental units of the SI system. This means that the size of the ampere is not fixed in terms of other units: we simply compare currents to a 'standard ampere'.

Household appliances, such as toasters and kettles, run on a current of a few amperes. An ampere is quite a sizeable flow of charge; however, especially in electronic circuits, we also often deal in milliamperes (mA, thousandths of an ampere, 10^{-3} A) or even microamperes (μ A, millionths of an ampere, 10^{-6} A).

If charge flows at a rate of one ampere, and continues to flow like that for a second, then the total amount of charge that has passed is one coulomb. This is how the size of the coulomb is fixed: in terms of the ampere and the second.

In practice it is probably best to picture coulombs of charge flowing at a rate of so many amperes. A current of 3 A, for example, is a flow rate of 3 coulombs of charge every second (3 C/s). With that

Worked example 3.1

120 C of charge passes in 1 minute. What is the current?

$$I = \frac{Q}{t} = \frac{120 \text{ C}}{60 \text{ s}} = 2 \text{ C/s} = 2 \text{ A}$$

KEY WORDS

galvanoscope *an instrument for detecting the presence of an electric current*

electron *a negatively charged particle that orbits round the nucleus of an atom*

ion *an atom bearing unequal numbers of electrons and protons*

conduction electrons *electrons in the conduction band of a solid, free to move under the influence of an electric field*

insulator *a material that resists the flow of electric charge*

current, therefore, it should be obvious that in 10 seconds a total of 30 C of charge will pass, or that to supply 12 C of charge the current must flow for 4 seconds.

You are strongly advised to understand the previous paragraph, rather than just remember. When you need the formula linking amperes, coulombs and seconds, however, it is:

$$Q = It \quad \text{Units}$$

Q coulombs (C)
I amperes (A)
t seconds (S)

In the formula, Q stands for the quantity of charge that passes when a current I flows for a time t .

Worked example 3.2

How long will a current of 5 A take to pass 100 C of charge?

$$Q = It, \text{ so } t = \frac{Q}{I} = \frac{100 \text{ C}}{5 \text{ A}} = 20 \text{ s}$$

Activity 3.1: Making a current tester

You can use a compass needle to make a simple **galvanoscope** (an instrument that detects the presence of an electric current). Simply wrap a few turns of thin insulated copper wire around a compass, connect the ends as shown in Figure 3.1 to a 1.5 V cell and a 1.5 V light bulb. When the wires are connected as shown, the bulb will light and the compass needle will move around in one direction. If you want to investigate this further, disconnect the cell, turn it round and reconnect it the other way round. The needle should move in the opposite direction.

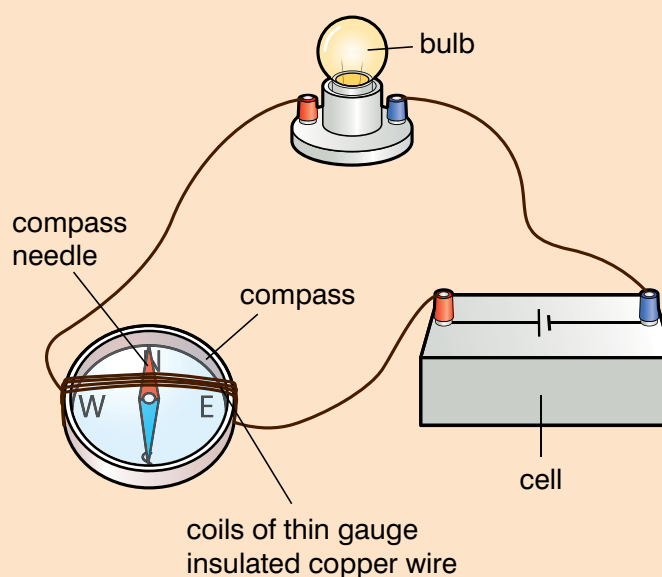


Figure 3.1 Current tester.

What are the charges that flow round a circuit?

To answer this question we must first look in rather more detail at the structure of an atom. All its positive charges are located in the central part, the nucleus. Each chemical element has a different number of positive charges in its nucleus, from one to nearly 100. Take copper as an example: it has 29 positive charges in the nucleus.

This means that an uncharged copper atom must have 29 negative electrons as well. They orbit round the nucleus, and each one has its own path. The first two electrons orbit in the innermost shell, the next eight fill the second shell and the following 18 just complete the third shell. That leaves a solitary electron as the first member of a new fourth shell.

It is this electron that accounts for the behaviour of copper as a conductor. It is comparatively easy to remove this electron, changing an uncharged copper atom into what we call a **copper ion** with an overall single positive charge (Figure 3.2).

In a copper wire the atoms are packed close together just as in any other solid. More precisely, what are packed together are the positive ions; the complete atoms except for those single outer electrons. They are there as well, so the metal as a whole is uncharged.

In each copper atom 28 of the electrons are still firmly bound in orbit around their nucleus, fixed in its place in the solid. The 29th electrons we call the **conduction electrons**. They remain trapped within the metal as a whole, but otherwise are free to drift about inside it.

They are the charges that move when an electric current flows down the wire. Being negative they will be repelled from the cell's negative terminal and attracted to the positive one (Figure 3.3).

All metals have these extra one or two outer electrons, so they will all conduct electricity. With most other elements, every electron without exception is tightly bound to its own nucleus, so most other elements are **insulators**.

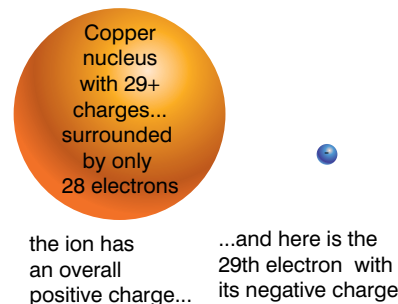


Figure 3.2 The copper atom can be changed into a positive copper ion plus a negative electron.

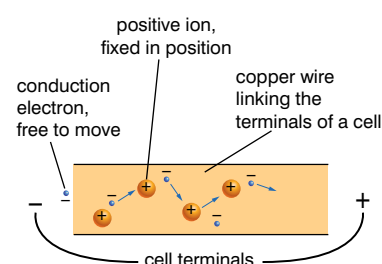


Figure 3.3 The unattached electrons in the copper wire move when it is connected to the terminals of a cell.

Conventional current

Electric currents have been investigated since the first cell was invented around 1800. It soon became obvious that charges were flowing round the circuit, but there was no way of telling whether they were positive charges going one way or negative charges going the other.

It was agreed to picture the flow as positive charges repelled from the positive plate, and

this was referred to as conventional current.

When the electron was discovered in 1895, it was realised that the guess had been wrong. However, so firmly fixed was the idea that even today we still mark on our circuits the 'conventional current' flowing from the positive pole of the cell to the negative.

You must understand, however, that the electron flow is really negative charge going the opposite way.

Activity 3.2: Testing conductivity

Not all materials have conduction electrons in their structure, so not all materials will conduct electricity. Use your simple galvanoscope (from Activity 3.1) to test which materials conduct electricity and which do not.

Connect your galvanoscope to a copper rod as shown in Figure 3.4a. Observe that the compass needle moves, showing that the rod conducts electricity, allowing an electric current to flow. Connect your galvanoscope

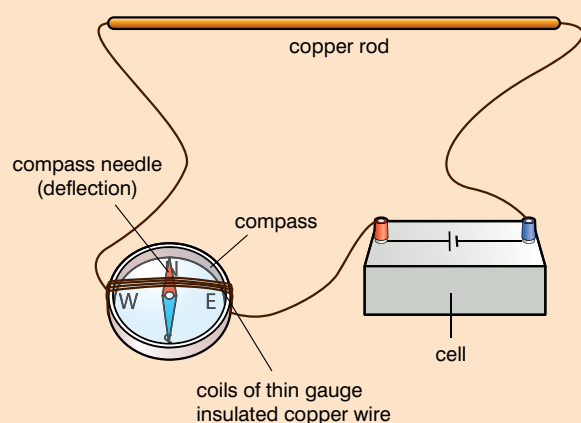


Figure 3.4a

to a glass or plastic rod, as in Figure 3.4b. Observe that the compass needle does not move, showing that the rod does not allow an electric current to flow. Repeat this for a selection of different materials of the same size, and rank your materials in order of the size of the deflection of the compass needle.

You will see that some materials conduct electricity better than others, while some materials do not conduct electricity at all.

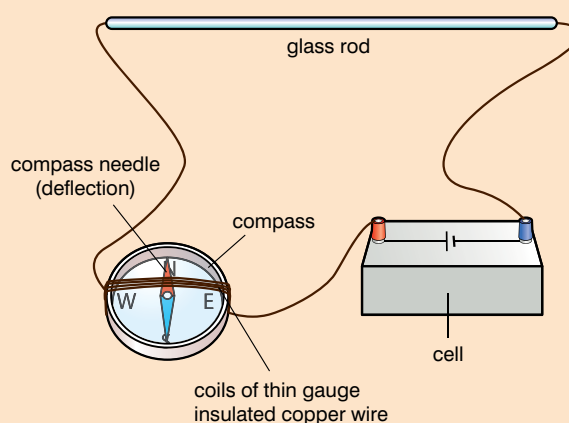


Figure 3.4b

KEY WORDS

drift speed *the average speed that an electron attains in an electric field*

Calculating the number of electrons that pass a point at a given length of time when the current in the wire is known

The wire shown in Figure 3.5 carries a current of I amperes. There are n free electrons per cubic metre of wire. Each electron carries a charge of e coulombs, the cross-sectional area of the wire is A square metres and the average **drift speed** of the electrons in this material is v metres per second.

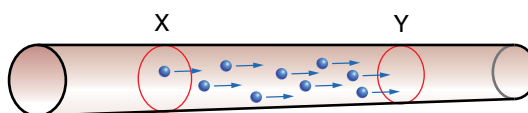


Figure 3.5

Assuming that it takes t seconds for an electron to pass from X to Y (and using the formula distance = speed $\times$ time), distance X–Y is vt metres. The volume of wire between X and Y is therefore Avt cubic metres.

The number of electrons between X and Y is therefore $nAvt$.

As each electron carries a charge of e coulombs, the total charge passing point Y in t seconds is $nAevt$.

The current, I , is the charge per second

$$I = \frac{nAevt}{t}$$

$$\text{or } I = nAev$$

As the charge carried by an electron is known to be 1.6×10^{-19} C, the number of electrons passing through a wire can be calculated if the current in the wire is known.

Worked example 3.3

A current of 5 A is flowing through a 2 mm diameter wire. The drift speed of electrons in the wire is 10^{-5} m/s and the charge on the electron is 1.6×10^{-19} C. Calculate the number of electrons passing a point in the wire in a second.

The cross sectional area (A) of the wire is πr^2 .

$$A = \pi \times \frac{d}{2} \times \frac{d}{2} = \pi \times 0.001 \times 0.001 = \pi \times 10^{-6} \text{ m}^2.$$

$$\text{Using } n = \frac{I}{Aev}$$

$$\begin{aligned} \text{number of electrons} &= \frac{5}{\pi \times 10^{-6} \times 1.6 \times 10^{-19} \times 10^{-5}} \\ &= 9.947 \times 10^{29} \end{aligned}$$

Study of electric charges in a metallic conductor

In Activities 3.1 and 3.2 you detected an electric current. Where does this electric current keep coming from and where does it keep going to? To start investigating the answer to this, look at Figure 3.6 which shows the top view of a corridor that ends up where it started. The whole place is filled with people who form a kind of endless queue. The door is shut, which means that nobody in the queue can move forward. Once the door is opened (once the circuit is switched on), all the people in that corridor can immediately start moving. They can keep circulating round and round the corridor, thus forming a continual current.

Of course when the door was opened the people filling the corridor found themselves free to move round the circuit, but they did not have to do so. Likewise charges will not circulate to give a current unless there is something that keeps 'pumping' them round. That is the job of the cell, as we shall see below.

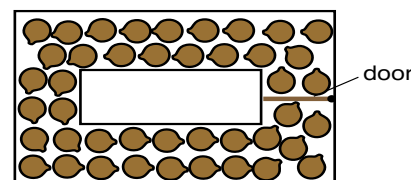


Figure 3.6

KEY WORDS

electrochemical cell *a device capable of deriving electrical energy from chemical reactions*

electrodes *conductors used to make contact with part of a circuit*

electrolyte *a solution that conducts electricity*

electromotive force *a source of energy causing current to flow in an electrical circuit*

e.m.f. *electromotive force*

dry cell *electrochemical cell containing electrolyte in the form of a paste*

polarisation *the formation of a film of hydrogen gas on the positive plate of a dry cell*

Electric energy from chemicals

In Activities 3.1 and 3.2 a flow of electric current is detected by a galvanoscope. What causes this current to flow around the circuit?

An electrochemical cell is one device that makes current flow round a circuit. Such cells have two **electrodes** made of two different **conductors**, an **electrolyte** solution (a solution that conducts electricity) which reacts with the electrodes, and a conductive wire through which electrons can flow.

Activity 3.3: Make your own electrochemical cell

Roll a lemon, orange, grapefruit, or other citrus on a firm surface to break the internal membranes.

Insert two rods – one copper, one zinc – into the fruit and connect as shown in Figure 3.7. Observe the size of the deflection of the needle on the instrument.

The electrodes in this cell are the copper rod and the zinc rod. The electrolyte here is the juice inside the orange.

Repeat the experiment using rods made from different materials and observe the deflection of the needle on the instrument. Do some pairs of materials produce a greater deflection than others?

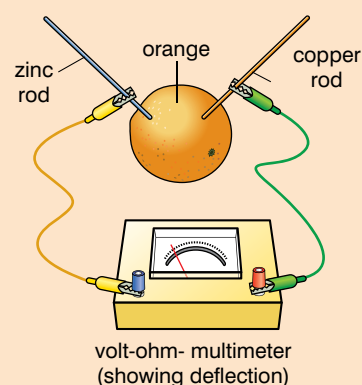


Figure 3.7

The volt

A volt is defined as the potential difference (p.d.) required to produce a current of 1 A in a circuit with a resistance of 1 ohm (or Ω). It is named after the Italian scientist Alessandro Volta (1745–1827).

Some cells are more powerful than others. An ordinary torch cell (usually referred to as a battery) is rated at 1.5 volts. We call this the **electromotive force** of the cell, usually shortened to **e.m.f.** It is the cell that pumps the charge round the circuit, and for the time being it is sufficient to think of the e.m.f. as its 'electrical pumping strength'. All other things being equal, the greater the e.m.f. of the cell the greater the current that it will drive round a circuit. You will have got the right idea when you do not like to hear people talking about volts flowing round a circuit. Volts do not flow. They cause coulombs of charge to flow round a circuit at a rate of amperes.

Sources of electricity

There are many different types of cell. Figure 3.8 illustrates the principles of one common form, which we call a **dry cell**. Experiment shows that the carbon plate is the positive one, the zinc is the negative, and the e.m.f. is very close to 1.5 V.

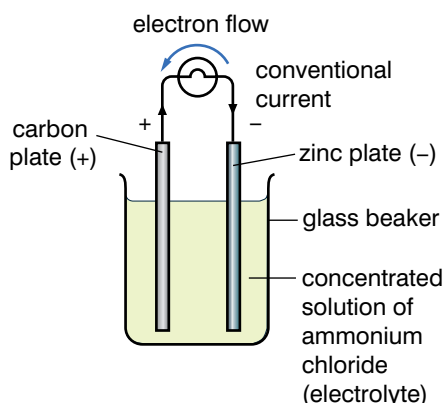


Figure 3.8 The principle of a common sort of cell.

Such a cell is about as portable as a glass of water, and we now see how it has been converted into the tough flashlight cell (commonly known as a battery) we are familiar with.

The negative zinc plate is shaped to form the casing of the cell (Figure 3.9). The carbon runs down the centre in the form of a rod (with a brass cap on the top, because carbon is easily damaged). The concentrated solution of ammonium chloride that has to go into the space between them is made much less runny by forming it into a paste or jelly. It is for this reason that the design is called a dry cell.

This leaves the complete cell as shown in Figure 3.10.

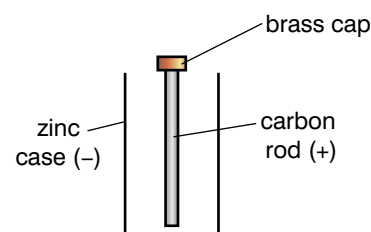


Figure 3.9 The zinc casing forms the negative plate.

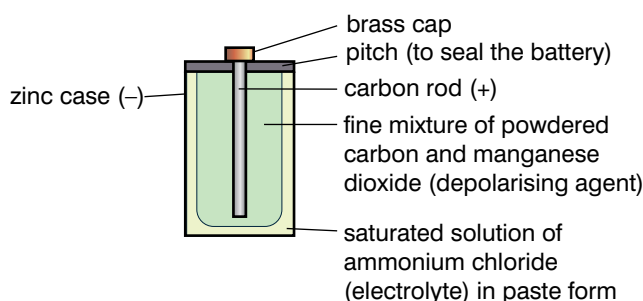


Figure 3.10 The complete dry cell.

The positive plate is both the carbon rod down the centre and the thick layer of powdered carbon that surrounds it.

Mixed in with the powdered carbon is manganese dioxide, another chemical in powder form. While the cell is giving a current it suffers from **polarisation**: the formation of a film of hydrogen gas on the positive plate which 'clogs' the cell up. The manganese dioxide is there to deal with this problem by supplying oxygen to convert this unwanted hydrogen to water. Sometimes the mixture of black powders is held in place round the central carbon rod with a cloth bag, but in other batteries it is just the 'stiffness' of the ammonium chloride paste that keeps it there.

KEY WORDS

primary cells *electrochemical cells that transform chemical energy directly to electrical energy*

secondary cells *electrochemical cells that have to be charged up by passing an electric current through them*

As the cell runs down, the zinc casing gets thinner and thinner, and when it gets very old the ammonium chloride may seep out from holes in the casing as a white corrosive paste.

There is nothing unique about carbon, zinc and ammonium chloride solution: any combination of two different metals (or one metal and carbon) placed in a solution of an electrolyte will produce a voltage. Other combinations are available, such as the alkaline cell, which uses potassium hydroxide for the electrolyte.

Primary cells and secondary cells

Cells that produce their voltage directly from chemical energy stored in the ingredients which make them up are known as **primary cells**. When all their chemicals have reacted, you have to buy a new one. The dry cell is one of these.

There are also **secondary cells**, which have first to be charged up by forcing a current 'backwards' through them. The commonest types of these are the increasingly popular rechargeable cells and the lead–acid accumulators that make up the battery in a car.

Activity 3.4: The direction of current flow

Part A

Connect the cell, motor and switch (open) as shown in Figure 3.11 on the edge of a table or bench. Clamp the motor to the edge of the table. Attach weights to the spindle of the motor, to hang over the edge of the table or bench. Close the switch. The motor will turn and the weights will move. Observe the movement of the weights; one weight will rise and one weight will fall.

Part B

Attach the cell the opposite way round. Close the switch and observe the movement of the motor and the weights. (x rises, y falls)

You will see that the motor turns in the opposite direction in Part B, causing the weight which rose in Part A to fall in Part B, and the one which fell in Part A to rise in Part B.

This demonstrates that the electrons in the circuit flowed in one direction in Part A and in the opposite direction in Part B.

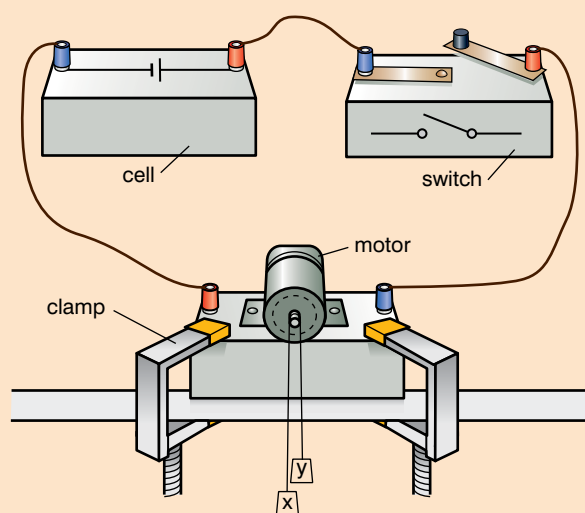


Figure 3.11

The 'standard ampere' – a note to turn back to when you have covered more physics

You may be puzzled as to how anyone can keep a standard ampere somewhere. That is not possible, of course, but we do have a way of defining it. (There is no need for you to remember the details.)

Two parallel wires each carry the same current (see Figure 3.12) and each current produces a magnetic field.

The magnetic field produced by one current-carrying wire acts on the other current-carrying wire to cause the **motor effect**, which pushes the wires apart.

The bigger the current in the wires, the greater the force pushing them apart.

If the wires are 1 metre apart and the push reaches 2×10^{-7} newtons on each metre length, then we say that each of the currents is one ampere.

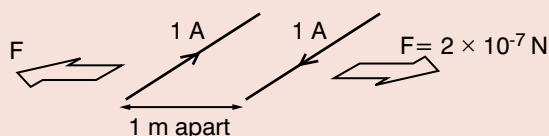


Figure 3.12

KEY WORDS

motor effect *when a current-carrying wire passes at right angles through a magnetic field, a force is exerted on the wire*

thermoelectricity *electricity produced when two metals of different temperatures are joined in a circuit*

Electric energy from heat

We have seen how electric energy can come from chemicals and we now see that it can also come from heat; this is called **thermoelectricity**.

If two different metals are joined in a circuit, and if one junction between the metals is hotter than the other, a very small voltage (a few millivolts, mV) is generated. This effect is known as the Seebeck effect after the German physicist Thomas Seebeck (1770–1831) who discovered it.

The circuit must be made from two different metals, any two, although some pairs may be better than others.

The thermocouple thermometer

The Seebeck effect is used in the thermocouple thermometer to measure temperature. It consists of a circuit made from any two different metals and includes a sensitive meter to measure small currents (Figure 3.13). If one of the junctions between the metals is hotter than the other, a small voltage is generated, which drives a tiny current round the circuit. The larger the temperature difference, the greater the current.

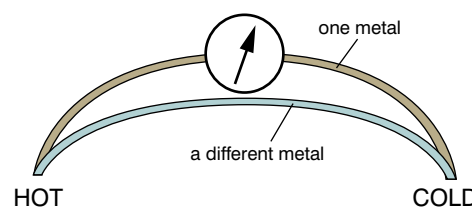


Figure 3.13 The thermocouple thermometer.

When using a thermocouple thermometer, it is usual to hold one junction at $0\text{ }^{\circ}\text{C}$ by keeping it immersed in melting ice. The other junction acts as the probe to investigate the temperature to be measured, as in Figure 3.14.

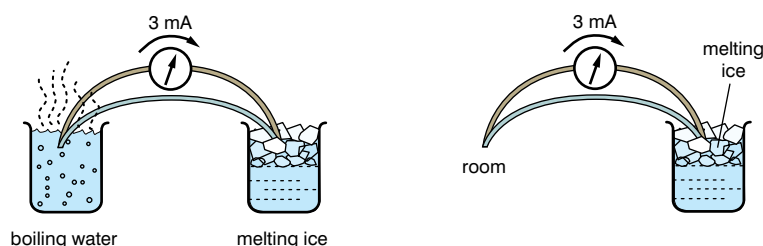


Figure 3.14 The thermocouple thermometer in use.

In the first part of Figure 3.14, a temperature difference of $100\text{ }^{\circ}\text{C}$ between boiling water and melting ice shows a current of 10 mA (milliamperes, thousandths of an ampere).

Each 1 mA therefore indicates a difference of $10\text{ }^{\circ}\text{C}$.

Thus when the probe is at room temperature and a current of 3 mA is measured, the junction at room temperature must be $30\text{ }^{\circ}\text{C}$ hotter than the cold junction, which is at $0\text{ }^{\circ}\text{C}$. Room temperature is therefore $30\text{ }^{\circ}\text{C}$.

Worked example 3.4

One junction of a thermocouple thermometer is immersed in melting ice and the other in boiling water. A current of 20 mA is recorded. The thermocouple thermometer is then used to measure the temperature of a liquid. One junction is immersed in melting ice and the other in the liquid whose temperature is to be measured. A current of 5.5 mA is recorded. What is the temperature of the liquid?

In the first, calibration, measurement, 20 mA represents a temperature difference of $100\text{ }^{\circ}\text{C}$. Therefore 1 mA represents a temperature difference of $5\text{ }^{\circ}\text{C}$. In the temperature measurement, the temperature difference between the junctions is $5.5 \times 5 = 27.5\text{ }^{\circ}\text{C}$.

The junction immersed in melting ice is at $0\text{ }^{\circ}\text{C}$. The temperature of the liquid being measured is therefore $0 + 27.5 = 27.5\text{ }^{\circ}\text{C}$.

Activity 3.5: Making a thermocouple thermometer

Create thermocouple junctions at both ends of a section of iron wire by twisting the ends together with copper wires.

Place one copper–iron junction in a beaker with ice water and leave the other junction outside. The two remaining ends of the copper wires should be connected to a sensitive galvanometer.

Heat the exposed junction with a Bunsen burner or match and record the current.

Does the current increase or decrease if the heat source is removed? Is the change in current immediate? Discuss these questions.

Summary

- Current is a flow of electric charge.
- Coulombs of charge flow at a rate of amperes.
- We need a voltage supply to cause the charge to circulate round the circuit.
- A primary cell uses the chemicals in it to supply electrical energy; a secondary cell has to be charged up first.

Review questions

- Charge flows along a wire at rate of 3 A. How many coulombs of charge will pass a given point in the circuit in:
 - 1 s
 - 12 s
 - 2 minutes?
- A very sensitive ammeter records a current of $30\ \mu\text{A}$. How long will it take before $6\ \mu\text{C}$ of charge has flowed through it?
- In an experiment to copperplate a coin, a current of 100 mA is passed through a solution of copper sulphate. If 30 C of charge must pass before the coin has enough copper deposited on it, for how long must you leave the current switched on?
- A single electron has a charge of $1.6 \times 10^{-19}\ \text{C}$. How many electrons must pass round the circuit of question 5 to achieve the 30 C?
 - For every two electrons that arrive at the coin which is being plated, one copper atom is deposited. If the mass of a copper atom is $1.1 \times 10^{-25}\ \text{kg}$, what is the total mass of the copper which now plates the coin?
- What name do we give to the unit 'ampere second' (A s)? What physical quantity would be measured in such a unit?
- You have two batteries, a large one and a tiny one, each consisting of a single dry cell. In terms of their performance, what would you expect to be the same for the two batteries and what would you expect to be different about them?
- What is the difference between a primary cell and a secondary cell?
- What temperature does the thermocouple in Figure 3.15 indicate for:
 - air
 - liquid B
 - liquid C?

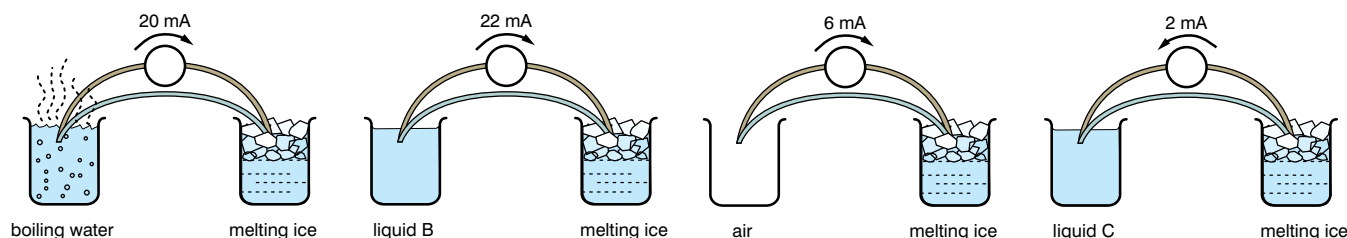


Figure 3.15

9. A thermocouple junction will respond to changes in the temperature of its surroundings more quickly than a mercury-in-glass thermometer would. There are two reasons for this:
 - a) the junction is smaller than the bulb full of mercury
 - b) the junction is not encased in glass.

Explain why each of these factors helps the thermocouple to respond quickly.
10. A metal wire is uncharged. Explain how it is possible for a current to flow through it.

3.2 Ohm's law and electrical resistance

By the end of this section you should be able to:

- Describe factors affecting the resistance of a conductor.
- Write the relationship between resistance R , resistivity ρ , length l and cross-sectional area A of a conductor.
- Calculate the resistance of a conductor using the formula $R = \rho l/A$.
- Find the relationship between resistivity and conductivity.
- Construct and draw an electric circuit consisting of a source, connecting wires, resistors, a switch and a bulb using their symbols.
- Explain why an ammeter should be connected in series with a resistor in a circuit.
- Explain why a voltmeter should be connected in parallel across a resistor in a circuit.
- Do experiments using an ammeter and a voltmeter to investigate the relationship between current and p.d. for metallic conductors at constant temperature.

Electrical resistance

The size of the current a cell will pump round a circuit depends on two things. One is the electromotive force, or e.m.f., of that cell, measured in volts. As we shall see in Section 3.4, adding a second cell **in series** (the current passes through one cell then the other) with the first makes a battery of cells, which has twice the e.m.f. of a single cell, and the charge will be pumped round the circuit at twice the rate. In other words, the current will double.

The second factor determining the current is the circuit through which the cell must drive the charge. Is it an easy circuit made of thick pieces of a good conductor, or is it a more difficult circuit consisting of thin wire made from a metal that does not conduct electricity so well? It is this second factor we are going to consider

now. Does the circuit have a low **resistance**, or does it have a high resistance?

We specify the resistance of a circuit by the number of volts of 'battery power' we would need to get a current of 1 A to flow round it. A low resistance circuit might need only 2 V per ampere (2 V/A), say. This suggests that a 1 V battery would produce a flow rate of 0.5 A round it, or that to get a current of 3 A going you would need to provide an e.m.f. of 6 V. A less easy circuit might have a resistance of 200 V/A; in other words, as many as 200 V would be needed to establish a current of just 1 A. It is important to study the behaviour of resistors in electrical circuits.

Measuring the resistance of a resistor

The principle is simple: apply a voltage across the resistor, and measure the size of the resulting current. A suitable circuit is shown in Figure 3.16 where R is the resistor to be measured.

Figure 3.16 also shows the symbol of a variable resistor (sometimes called a 'rheostat'). By moving a slider or rotating a knob you can alter its resistance. This in turn will alter the total resistance of the whole circuit, and will therefore control the current drawn from the battery which then passes through the ammeter.

In a lighting circuit a variable resistor would act as a dimmer switch to make the bulb fainter or brighter.

The ammeter may be placed anywhere in the circuit since the same current flows all the way round. The voltmeter is not part of the circuit itself; it is placed alongside to measure the drop in voltage between the two ends of R . The variable resistor is not essential, but it enables you to alter the voltage drop across R and the current through it so as to get check readings.

If you try this activity for yourself, you should choose a resistor to measure that is not significantly warmed by the current you send through it. A length of resistance wire open to the air should do, but avoid using a light bulb. You will find out why in the next section.

Your results should be recorded in a table like that in Figure 3.17.

KEY WORDS

in series wiring an electrical circuit so that there is only one path for the current to take between any two points

resistance the opposition to a flow of current in an electrical circuit

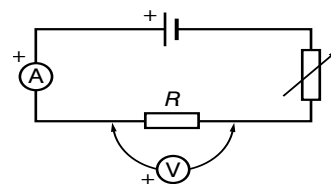


Figure 3.16 Measuring the resistance of a resistor.

p.d. V across R (volts)	current I through R (amperes)	$R = \frac{V}{I}$ (ohms)
Average		

Figure 3.17

KEY WORDS

Ohm's law *the current that flows through a conductor is proportional to the potential difference between its ends*

The ohm (Ω)

The unit for resistance is the ohm (Ω).

It is named after Georg Ohm (1787–1854), a German physicist who was one of the first to investigate how currents flowed in circuits.

The abbreviation for ohm should really be a capital 'O', but that could be confused with a zero. Luckily there is a letter in the Greek alphabet called 'omega', which provides the ideal replacement: the abbreviation for ohms, therefore, is ' Ω ', which is a capital omega.

Worked example 3.5

A 12 V car battery is connected to a circuit for which total resistance is 6 Ω . What current will flow?

$$I = \frac{V}{R} = \frac{12}{6} = 2 \text{ A}$$

As you repeat the experiment for different values of V and I , you should find that the value for R remains constant. This illustrates **Ohm's law**.

Ohm's law

For a metal wire at a constant temperature, the current that flows through it is proportional to the potential difference (the voltage drop) between its ends.

In other words, if the voltage drop across the wire doubles, charge will flow through it at exactly twice the rate (that is, the current doubles too). Put yet another way, $\frac{V}{I}$ stays constant: the wire's

resistance does not change. Notice that this will apply only if the temperature of the conductor does not change.

This can be put into a formula. The current I that is produced when a battery of e.m.f. V is connected to a circuit of resistance R is given by:

$$I = \frac{V}{R}$$

Current (A) I ampere

Voltage (V) V volt

Resistance (Ω) R ohm

To make the current larger you could either increase V by adding another cell or you could reduce the resistance of the circuit.

Worked example 3.6

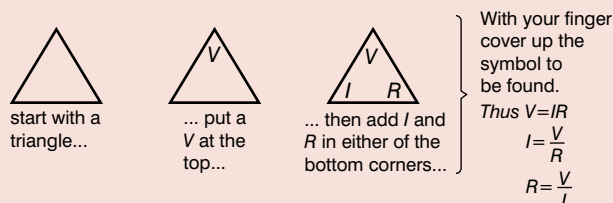
A 12 V battery is connected to a circuit. If the current is 2 mA, what must the resistance of the circuit be? (Notice here that the current is given as 2 mA. Before it can be entered into the formula it must be converted to 0.002 A (or to 2×10^{-3} A).)

$$I = \frac{V}{R}, \text{ so rearranging we get } R = \frac{V}{I} = \frac{12}{0.002} = 6000 \text{ } \Omega$$

A thousand ohms is one kilohm (rather than 'kiloohm'), so the answer could be expressed as 6 k Ω .

A trick to help you with the calculations

If you have had a problem with changing $I = \frac{V}{R}$ into $R = \frac{V}{I}$, there is a method which will help. The diagram shows you how.



Provided you remember that V goes at the top, you will not go wrong.

Worked example 3.7

What voltage battery would be needed to send a current of 3 A round a circuit for which the total resistance is 4 Ω ?

Here $I = 3$ A and $R = 4$ Ω .

$$V = IR = 3 \times 4 = 12 \text{ V}$$

Factors affecting the resistance of a conductor

Effect of heat on resistance

If you display the current and voltage readings measured when studying a resistor at constant temperature (recorded in a table such as Figure 3.17) in the form of a graph of the current through the resistor plotted against the voltage drop applied across it, you should get a straight line (Figure 3.18). Double the potential difference (p.d.) across the wire, and you will double the current flowing through it.

However, if you use a light bulb as your resistor and take a range of readings such that the filament of the bulb is varied from not even red-hot to brilliantly white-hot, the graph of your readings will form a curve (Figure 3.19). As you increase the voltage and the lamp glows more brightly, the current does not rise as rapidly as expected.

As the filament becomes white hot, its temperature increases by at least several hundred degrees. The resistance of metals rises with temperature, and that is why a hot bulb does not conduct as well as a cool one.

Instead of taking current and voltage readings, it is useful to measure the resistance of a light bulb at as wide a range of currents as possible. At one extreme, readings should be taken when the current is so small that the filament is not even glowing feebly red; at the other extreme the bulb should be brighter than normal.

Use these readings to plot a graph of resistance against current, as in Figure 3.20 on the next page. If the temperature had not changed, the resistance would have been constant (line (a)). As it is, the resistance rises markedly (line (b)).

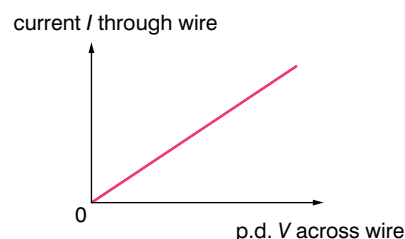


Figure 3.18 A graph of current against voltage drop for a resistor, R .

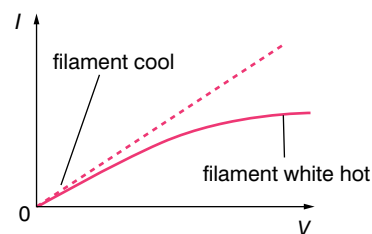
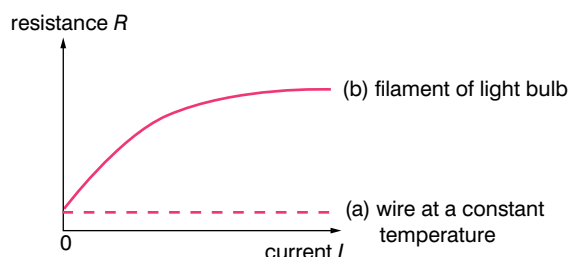


Figure 3.19 As the temperature of the filament rises, so does its resistance.



Worked example 3.8

A copper resistor ($\alpha = 4.3 \times 10^{-3} \text{ K}^{-1}$) whose value is $10.0 \, \Omega$ at $0 \, ^\circ\text{C}$ is warmed from that temperature up to $150 \, ^\circ\text{C}$. Work out its resistance when hot.

Increase in resistance

$$= \alpha R_0 \Delta T$$

$$= 4.3 \times 10^{-3} \times 10 \times 150$$

$$= 6.45 \, \Omega$$

Resistance when hot

$$= 10 + 6.45 = 16.45 \, \Omega$$

Figure 3.20 Resistance–current graph.

Notice that the line will not pass through the origin. If you continue the curve back so it approaches or equals zero, you will have the resistance of the bulb when it is cold (that is, at room temperature). At its working temperature the resistance of the bulb may well have risen to ten times that value.

The resistance of a metal increases in an approximate straight line if you plot it against its temperature. The increase in the value of the resistance as it warms up depends on:

- The rise in the wire's temperature, ΔT . The greater the rise, the bigger the increase.
- How many ohms of resistance it possesses. Since this will vary with its temperature, we work on the basis of its resistance at $0 \, ^\circ\text{C}$ and call it R_0 . The greater the number of ohms at the start, the greater the increase in resistance will be.
- The metal it is made from.

Putting these together, we can say:

$$\text{the increase in resistance} = \alpha R_0 \Delta T$$

Alpha (α) is a constant, which varies from one metal to another. We call it the metal's temperature coefficient of resistance. The units of α have to be K^{-1} (kelvin $^{-1}$) in order to give an answer that is in ohms. You will probably be working in degrees Celsius rather than kelvin, but there is no need to worry, remember that a rise in temperature of $10 \, ^\circ\text{C}$ is exactly the same as a rise of $10 \, \text{K}$.

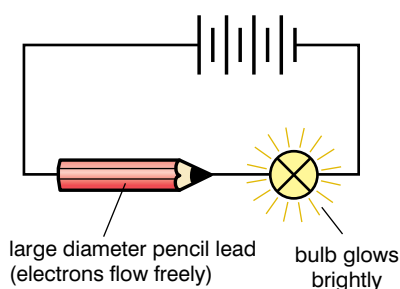


Figure 3.21 A large diameter pencil lead, bright bulb

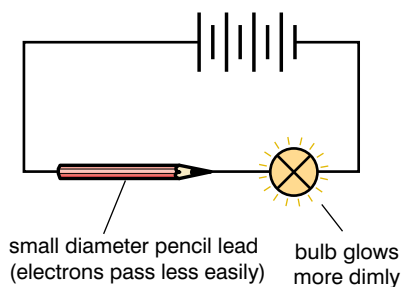


Figure 3.22 A small diameter pencil lead, dim bulb

Effect of length and diameter of resistor on resistance

Connect a light bulb (1 to 2 V) in series with a pencil lead (pencil 'lead' is made of carbon, an element frequently used in resistors) and a 6 V battery (four 1.5 V cells connected in series). Turn off the room lights and observe the brightness of the bulb.

Move one wire contact along the *length* of the pencil lead and observe the change in the intensity of light. The light should become brighter as the resistor becomes shorter, as there is less resistance impeding the circuit.

Repeat the activities with a pencil lead of different *diameter* and observe the changes in the intensity of light. The light should become brighter when the diameter of the resistor is larger as it is easier for the current to flow.

The relationship between resistance R , resistivity ρ , length l and cross-sectional area A of a conductor

The resistance of a metal wire at a given temperature is determined by three factors:

- Its length l , in metres – the resistance is proportional to l , so if the length doubles so does the resistance.
- Its area of cross-section A , in m^2 – the resistance is inversely proportional to A , so a wire with twice the cross-sectional area will have only half the resistance.
- The material from which the wire is made – copper, for example, is a better conductor than iron.

Thus the resistance R of a wire can be expressed in the form:

$$R = \frac{\rho l}{A}$$

The symbol ρ (the Greek letter rho) is a constant, the value of which depends on the material from which the wire is made – the value for iron will be greater than the value for copper. We call ρ the resistivity of the material: it is defined by the equation above.

The units of resistivity are $\Omega \text{ m}$ – ohms multiplied by metres.

To see this, consider the units of the right-hand side. They will be $\Omega \text{ m} \times \text{m}$ divided by m^2 , which works out correctly to give the resistance in ohms.

Worked example 3.9

What is the resistance of a copper cable that has a cross-sectional area of 1 cm^2 and a length of 2 km ? The resistivity of copper is $2 \times 10^{-8} \Omega \text{ m}$.

Be careful over the units.

$$l = 2 \text{ km} = 2 \times 10^3 \text{ m}$$

$$A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2 \text{ (since there are } 100 \times 100 \text{ centimetre squares in a metre square)}$$

$$R = \frac{\rho l}{A}$$

Putting in the values, we get

$$R = \frac{2 \times 10^{-8} \times 2 \times 10^3}{1 \times 10^{-4}} = 0.4 \Omega$$

A tip for you

When you come to tackle a question on this topic, especially in an exam, you may have a moment of doubt: is the expression $R = \frac{\rho l}{A}$, or is it $R = \frac{\rho A}{l}$?

If you remember that the units of ρ are $\Omega \text{ m}$, you can quickly check that the second expression would give units for R of $\Omega \text{ m}^2$, which is wrong.

The relationship between resistivity and conductivity

The resistivities of most metals are in the range 10^{-7} to $10^{-8} \Omega \text{ m}$. Those with larger resistivities conduct electricity less well.

The resistivity of an insulator such as dry polythene may be as high as $10^{15} \Omega \text{ m}$.

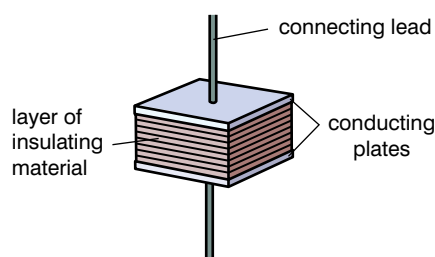


Figure 3.23 The capacitor.

KEY WORDS

semiconductors *materials that have an electrical conductivity between that of a conductor and an insulator*

ammeter *an instrument used to measure the electric current in a circuit*

series circuit *an electrical circuit in which the current passes through each point in turn*

voltmeter *an instrument used to measure potential difference in a circuit*

potential difference *the difference in electrical charge between two points in a circuit measured in volts*

Conductors and insulators are two extremes, but there are only a few materials in between these extremes: germanium at room temperature, for instance, may display a resistivity of around $0.001 \, \Omega \, \text{m}$. We call such materials **semiconductors**.

The capacitor (Figure 3.23) is an electrical component made of a combination of resistor and insulator. Many circuits use the capacitor to store and release charge. While there are many types of capacitor, they all have the same basic features – two conducting plates separated by a layer of insulating material (see page 71).

Constructing and drawing electric circuits

An electric current is a flow of charge. A circuit is made of conducting materials, with a device such as a cell, which keeps pumping the charges round the circuit.

Figure 3.22a shows the symbol for a cell. We often refer to it as a battery, though strictly speaking this is incorrect: 'battery' is the term reserved for a whole collection of cells acting together. The plus and minus signs are not usually marked on the drawing, so you must remember which is which. Remember too that **conventional current flows round the circuit from positive to negative**.

Figure 3.22b represents a battery of four cells. Notice that in order for them all to be pumping the charge the same way, the '+' terminal of one cell has to be joined to the '-' terminal of the next one.



Figure 3.24 a) The symbol for a cell; b) a battery of four cells.

The symbols for a cell, a light bulb, an ammeter and a switch are shown in the circuit in Figure 3.25. The **ammeter** is an instrument that measures the size of a current. It is fitted into the circuit so the current to be measured flows through it. In order that the ammeter does not reduce the current it has been put in to measure, it is essential that the instrument allows the current to flow through it freely (has a low resistance).

It is important that you connect the ammeter the correct way around, otherwise the current would try to twist it backwards. The terminal on it marked '+' needs to be joined to the positive side of the battery, as shown in Figure 3.25.

(Sometimes the terminals are colour-coded. The '+' terminal will be red, and the '-' terminal black.)

Figure 3.26 shows the symbols for a range of electrical components. If you have access to equipment, test what you have learnt so far by using some of these components to make circuits such as that shown in Figure 3.25.

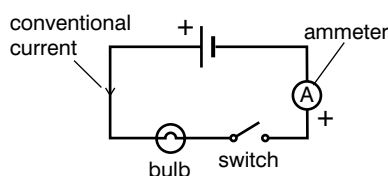


Figure 3.25 A simple circuit.

Try constructing the circuit shown in Figure 3.16 (page 93). Use the ammeter and **voltmeter** to make measurements, varying the current using the variable resistor. Record your measurements and discuss your findings with the class.

If you have access to other equipment – such as resistors, variable resistors, ammeter, voltmeter, capacitors – use these and observe what happens when you construct a circuit. Using the symbols for each component, draw a circuit diagram.

The correct position for an ammeter in a circuit

Do you think the current would be the same both before and after the light bulb in Figure 3.25? We will investigate this for the slightly more complex circuit of Figure 3.27 where there are two bulbs.

We call a circuit like this a **series circuit**: the current has to pass first through one bulb and then through the other. Notice again the way the three ammeters have been connected, with their '+' terminals nearer the positive side of the battery.

The result may surprise you:

the current has the same value all the way round a series circuit.

The correct position for a voltmeter in a circuit

Consider the circuit of Figure 3.28, which shows two resistors in series in a circuit. What is the voltmeter doing? You may think of the voltmeter as being connected in parallel (see Section 3.3 for a description of parallel circuits) with the first resistor. This is true, but it is probably not the best way of looking at it.

Unlike an ammeter, the voltmeter is not part of the circuit. If you wish to measure a voltage drop, you place the voltmeter on the bench nearby. One lead comes from the voltmeter to see what conditions are like at one point in the circuit, while the other lead goes to a second point in the circuit to find out what things are like there: the voltmeter then indicates the difference between the two points. We call it the **potential difference** (p.d.) between the ends of the resistor, and measure it in volts. Sometimes we speak of it as the voltage drop (or just the voltage) across the resistor.

The voltmeter shows the difference in something – but the difference in what? It might help you to think of a comparison with heat: a temperature difference makes thermal energy flow, and a potential difference makes charge flow.

Ideally a voltmeter should draw no current from the circuit it is investigating. It must therefore have a very high resistance.

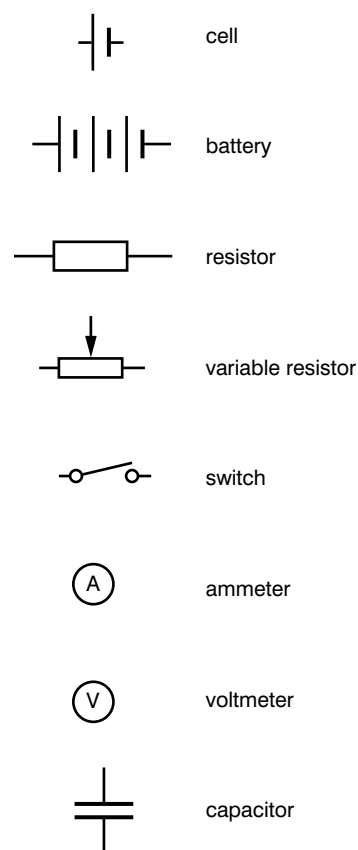


Figure 3.26 Some circuit symbols.

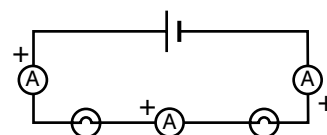


Figure 3.27 Testing the current before and after it passes through the light bulbs.

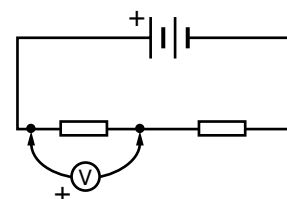


Figure 3.28 A voltmeter connected in a circuit.

Summary

- In a series circuit, the current is the same all the way round.
- You calculate current using $I = \frac{V}{R}$.
- You can measure the value of a resistor with an ammeter and voltmeter by applying Ohm's law.
- The resistance of a metal increases as its temperature rises. Its temperature coefficient of resistance (α) provides a numerical measurement of this.
- You can work out the resistance of a wire from $R = \frac{\rho l}{A}$, where ρ is the resistivity of the material of the wire.

Review questions

1. A battery of e.m.f. 6 V is connected to a circuit with a total resistance of 12 Ω . What current would you expect to flow?
2. A flashlight bulb has '2.4 V 0.3 A' marked on it. This means 'If you connect it to a 2.4 V battery a current of 0.3 A will be driven through it; this will make it white-hot'.
 - a) Estimate the resistance of that flashlight bulb when it is lit up.
 - b) If the bulb was connected to the mains (either 110 V or 220 V) instead, would the current initially be 0.3 A, more than 0.3 A or less than 0.3 A? Give your reason.
 - c) What would happen next? Explain.
3. The resistivity of iron is $1.0 \times 10^{-7} \Omega \text{ m}$. Find the resistance of a 12 km length of a railway line with a cross-sectional area of 200 cm².
4. A tungsten resistor ($\alpha = 5.8 \times 10^{-3} \text{ K}^{-1}$) with a value of 10.0 Ω at 0 °C is warmed from that temperature up to 150 °C. Work out its resistance when hot.
5. Figure 3.29 shows a circuit with a cell, a variable resistor and a lamp.

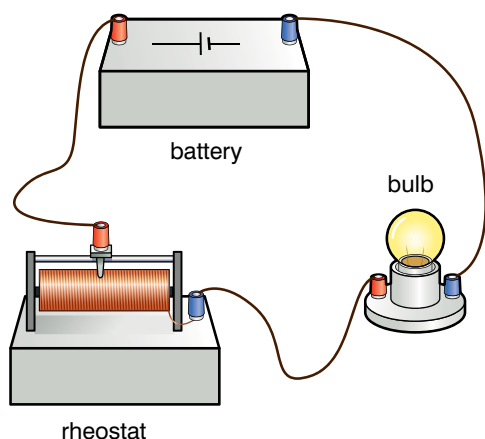


Figure 3.29

- a) Draw a circuit diagram of this arrangement.
- b) Redraw the circuit diagram adding an ammeter to measure the current in the circuit, and a voltmeter to measure the p.d. across the lamp.
- c) Describe how you would use this circuit to measure the current through the lamp for various p.d.s across it and describe the shape of a graph of p.d. against current that would be obtained.
- d) What would you change in this circuit to demonstrate Ohm's law?

6. A student designs the circuit shown in Figure 3.30 in order to try to test Ohm's law.
- What is wrong with this circuit?
 - Draw the circuit diagram the student should have used.
7. What is the difference between a cell and a battery?

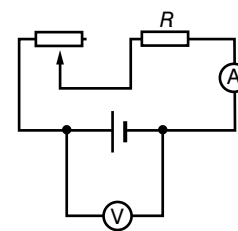


Figure 3.30

KEY WORDS

resistors *electrical components that limit or regulate the flow of electrical current in a circuit*

3.3 Combinations of resistors

By the end of this section you should be able to:

- Identify combinations of resistors in series, parallel and series-parallel connection.
- Derive an expression for the effective resistance of resistors connected in series.
- Derive an expression for the effective resistance of resistors connected in parallel.
- Calculate the effective resistance of resistors connected in series.
- Calculate the effective resistance of resistors connected in parallel.
- Calculate the current through each resistor in simple series, parallel and series-parallel combinations.
- Calculate the voltage drop across each resistor in simple series, parallel and series-parallel connections.

Resistors in different combinations

We shall now look at **resistors** connected in electric circuits in different ways – in series (Figure 3.31) and in parallel (Figure 3.34). In both cases, it might help to think of an electrical circuit as a large crowd of people trying to get into a stadium.

In Figure 3.29 overleaf, two resistors – $1\ \Omega$ and $3\ \Omega$ – are connected in series. Think of a line of people trying to get into a stadium, they have to queue up to get through one gate ($1\ \text{ohm}$) and then through a narrower one ($3\ \text{ohms}$) on their way into the stadium.

In Figure 3.35 on page 104 three resistors – $2\ \Omega$, $3\ \Omega$ and $6\ \Omega$ – are connected in parallel. Think of this as the queue for the stadium again. In this case, the stadium has three (differently sized) gates for people to use, and they can enter the stadium much more easily.

Resistors connected in parallel are thus seen to have a lower total resistance than resistors connected in series.

In practice, electrical circuits can be quite complicated and can consist of resistors and other electrical components connected both in series and parallel.

Activity 3.6: Series and parallel circuits

Do these activities in groups.

Connect one bulb in series with a battery and note its brightness. Now connect a second and third bulb in series with the bulb.

Is there any change in the brightness of the first bulb when the second and third bulbs are added?

Once all bulbs are lit, remove one of the bulbs. What happens to the brightness of the others?

Connect one bulb to a battery and note its brightness. Now connect a second and third bulb in parallel with this bulb.

Is there any change in brightness of the first bulb when the second or third ones are added?

Once all bulbs are lit, remove one. Is there any change in the brightness of the remaining bulbs?

Series combination

Look at the circuit in Figure 3.31. There are two resistors ($1\ \Omega$ and $3\ \Omega$) in series. Each resistor offers a different resistance to the current flow and there is therefore a different voltage drop across each resistor.

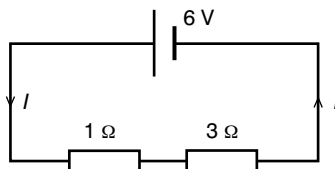


Figure 3.31 Resistors in series in a circuit.

With resistors in series, it is easy to work out the resistance of the circuit as a whole. You simply add up the separate values. In this case the circuit has a resistance of $4\ \Omega$. (Be sure you could then work out that a current of $1.5\ \text{A}$ will be drained from the battery, as $I = \frac{V}{R} = \frac{6}{4}$ from Ohm's law.)

For a series circuit, the total resistance $R = R_1 + R_2$

where R_1 and R_2 are the resistances of the separate components.

Worked example 3.10

Consider the circuit shown in Figure 3.32.

- Calculate the total resistance of the whole circuit.
- Calculate the current in the circuit.

Since the circuit is a series one, the same current must flow through all three resistors. Consider the $3.5\ \Omega$ resistor.

- Calculate the voltage drop necessary to send $2\ \text{A}$ through $3.5\ \Omega$.
- Calculate the p.d. across the other two resistors.

You will need to use the relationship $R = \frac{V}{I}$ in the form of $V = IR$ and $I = \frac{V}{R}$.

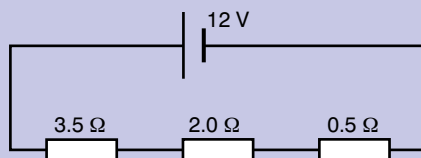


Figure 3.32

- Total resistance in the circuit (series circuit)
 $= 3.5 + 2 + 0.5 = 6\ \Omega$

- Voltage supplied to the circuit is $12\ \text{V}$

Resistance in the circuit is $6\ \Omega$

From Ohm's Law, $I = \frac{V}{R}$

$$I = \frac{12}{6} = 2\ \text{A}$$

c) Resistance = $3.5 \, \Omega$

Current = $2 \, \text{A}$

$$V = IR$$

$$V = 2 \times 3.5 = 7 \, \text{V}$$

The voltage drop across the $3.5 \, \Omega$ resistor is $7 \, \text{V}$.

d) For the $2 \, \Omega$ resistor:

resistance = $2 \, \Omega$, current = $2 \, \text{A}$

$$V = IR = 2 \times 2 = 4 \, \text{V}$$

For the $0.5 \, \Omega$ resistor:

resistance = $0.5 \, \Omega$, current = $2 \, \text{A}$

$$V = IR = 0.5 \times 2 = 1 \, \text{V}$$

Worked example 3.11

Calculate the size of resistor X in Figure 3.33.

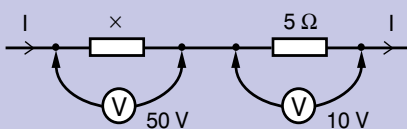


Figure 3.33

First look at the $5 \, \Omega$ resistor. From $I = \frac{V}{R}$, a p.d. of $10 \, \text{V}$ across it must mean that a current I is flowing through it, given by:

$$I = \frac{V}{R} = \frac{10}{5} = 2 \, \text{A}$$

Now look at resistor 'X'. Since they are in series the same $2 \, \text{A}$ is flowing through it. To achieve this there is a p.d. of $50 \, \text{V}$ across it.

$$R = \frac{V}{I} = \frac{50}{2} = 25 \, \Omega$$

Parallel combination

It is less obvious how to work out the overall resistance of a circuit that consists of two resistors, not in series but in parallel (Figure 3.34).

If the two resistors are R_1 and R_2 , then they behave like a single resistor R the value of which is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

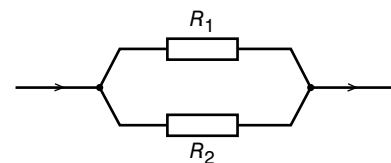


Figure 3.34 Resistors in parallel.

Worked example 3.12

A circuit consists of a $2\ \Omega$ resistor and a $3\ \Omega$ resistor in parallel. A $3\ \text{V}$ battery is connected to the circuit. How big a current is drawn from it?

We must first calculate the effective resistance of the circuit using the formula for resistors in parallel.

Here,

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{2} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

That gives us 'one over R ', so to find R we must turn it over.

$$R = \frac{6}{5} = 1.2\ \Omega$$

The question now becomes 'What current will a $3\ \text{V}$ battery send round a $1.2\ \Omega$ circuit?'

$$I = \frac{V}{R} = \frac{3}{1.2} = 2.5\ \text{A}$$

The voltage drop across resistors in parallel

Look at the circuit of Figure 3.35. Remembering that there is no voltage drop down a conducting lead, you should be able to see that the left-hand end of all three resistors is at $+6\ \text{V}$ and their right-hand ends are at $0\ \text{V}$.

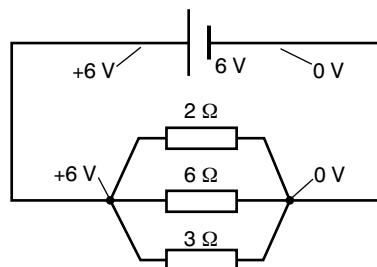


Figure 3.35

That illustrates what happens in this case:

if resistors are connected in parallel, they all have the same voltage drop across them.

This provides us with a means to work out the current in each branch of the circuit. Take the $2\ \Omega$ resistor as an example. The full $6\ \text{V}$ of the battery is dropped across it, so the current I through it is given by:

$$I = \frac{V}{R} = \frac{6}{2} = 3\ \text{A}$$

Worked example 3.13

For the circuit shown in Figure 3.36:

- calculate the voltage drop across each resistor
- calculate the current through each resistor
- calculate the effective resistance of the circuit.

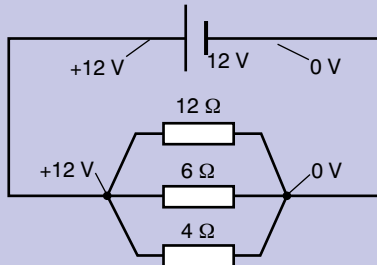


Figure 3.36

- As the resistors are connected in parallel, the voltage drop will be the same across each, and will be equal to the full 12 V of the battery.

- The voltage across the 12 Ω resistor is 12 V,

$$I = \frac{V}{R} = \frac{12}{12} = 1 \text{ A}$$

The voltage across the 6 Ω resistor is 12 V,

$$I = \frac{V}{R} = \frac{12}{6} = 2 \text{ A}$$

the voltage across the 4 Ω resistor is 12 V,

$$I = \frac{V}{R} = \frac{12}{4} = 3 \text{ A}$$

- Total resistance of the circuit is R .

$$\frac{1}{R} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4}$$

$$= \frac{1 + 2 + 3}{12} = \frac{6}{12} = \frac{1}{2}$$

$$R = 2 \Omega$$

Worked example 3.14

Two resistors of 10 Ω and 15 Ω are connected. What is their combined resistance if they are connected:

- in series
- in parallel?

- In series

$$\begin{aligned} R &= R_1 + R_2 \\ &= 10 + 15 = 25 \Omega \end{aligned}$$

- In parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{10} + \frac{1}{15}$$

$$= \frac{3 + 2}{30} = \frac{5}{30}$$

$$R = \frac{30}{5} = 6 \Omega$$

Summary

- Resistors in series simply add.
- Resistors in parallel add by a ' $\frac{1}{R}$ ' formula. Two resistors in parallel will conduct better than either one on its own.
- The resistance of the whole circuit includes the resistance within the battery itself, and this may not always be negligible.

- Where the circuit branches in a parallel circuit, the sum of the currents approaching the junction equals the sum of the currents leaving it.
- In a series circuit, the separate voltage drops (potential differences) across all the resistors add up to the voltage of the battery.
- In a parallel circuit each branch will have the same voltage drop across it.

Review questions

1. What should ammeters (a), (b) and (c) read in Figure 3.37?

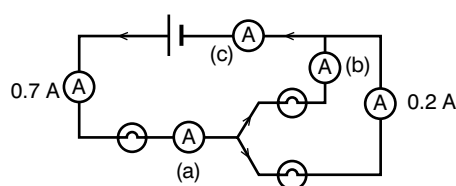


Figure 3.37

Which bulb should be the brightest? Why?

2. Give the missing ammeter readings a and b in Figure 3.38. Suggest why more current flows through some bulbs than through others.

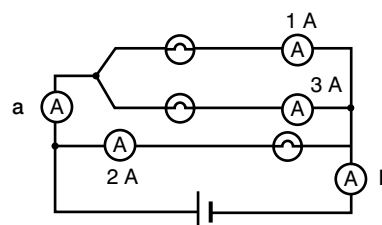


Figure 3.38

3. Give the missing ammeter readings c and d in Figure 3.39. Assume all the bulbs are identical ones. How would you expect the brightness of the different bulbs to compare?

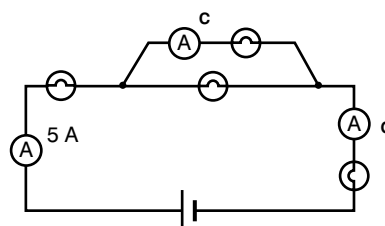
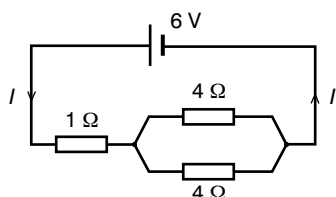
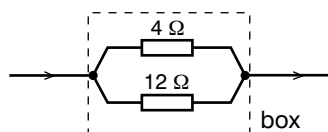


Figure 3.39

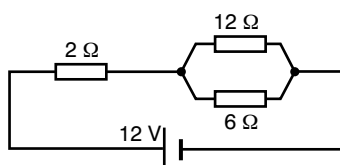
4. Draw a circuit diagram to show a battery of two cells with three light bulbs connected in parallel. Include two switches: one must turn one of the bulbs on and off, and the other must do the same for the other two bulbs together.
5. For the circuit diagram in Figure 3.40:
 - a) What is the effective resistance of the two $4\ \Omega$ resistors in parallel?
 - b) What is the total resistance of the circuit?
 - c) Work out the current I .

**Figure 3.40**

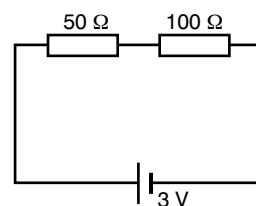
6. What single resistor could be placed in the box to replace the two shown in Figure 3.41?

**Figure 3.41**

7. Calculate the effective resistance of a $25\ \Omega$ resistor and a $100\ \Omega$ resistor in parallel. What current will be drawn from a $50\ \text{V}$ supply connected to the circuit?
8. a) Work out the total resistance of the circuit in Figure 3.42.
b) What current will be drawn from the battery?

**Figure 3.42**

10. In the circuit of Figure 3.43 the battery has negligible internal resistance. Calculate:
 - a) the total resistance in the circuit
 - b) the current flowing in the circuit
 - c) the potential difference across the $50\ \Omega$ resistor.

**Figure 3.43**

3.4 E.m.f. and internal resistance of a cell

By the end of this section you should be able to:

- Define the electromotive force (e.m.f.) of a cell.
- Distinguish between e.m.f. and terminal p.d. of a cell.
- Write the relationship between e.m.f., p.d., current and internal resistance in a circuit.
- Use the equation $V = E - Ir$ to solve problems in a circuit.
- Identify cell combinations in series and parallel.
- Compare the e.m.f. of combinations of cells in series and parallel.

The electromotive force of a cell

As we saw in Section 3.1, a cell pumps charge round a circuit. Its effectiveness at doing this is called its electromotive force (e.m.f.), in volts. A voltmeter is an instrument that can measure the e.m.f. of a cell: it has two leads going to it, and one is connected to each terminal of the cell.

Volts do not flow round a circuit. Volts cause coulombs of charge to flow at a rate of amperes. If we know the resistance R of a circuit, we can predict the current I that a battery will send round it, from $I = \frac{V}{R}$.

The resistance inside a cell

Suppose you short-circuit a cell. This means that you join its two terminals by a circuit that effectively has no resistance – a short piece of very thick copper wire, for instance. The cell has an e.m.f. V , but the circuit apparently has no resistance R . What happens then? Does the current increase without limit?

The point we are forgetting is that the cell has to pump the charge round the whole circuit, and that includes the part within the cell as well as the external circuit. Internal resistance varies between different cells, but it is what finally sets a limit to the current a cell can supply.

A 1.5 V torch cell (represented in Figure 3.45 by the dotted line) typically has an internal resistance of up to an ohm. This means that even if you short-circuit the cell, there is still an ohm of resistance. The biggest current it can deliver is given by $I = \frac{V}{R} = \frac{1.5}{1} = 1.5$ A.

So far, we have taken it for granted that if you double the e.m.f. in a circuit, you will double the current. This should be checked experimentally, but it needs some careful thought in order to give it a fair test.

The obvious thing to do is to add cells one at a time to a circuit, measure the current and see if it increases accordingly. A fair test

Activity 3.7: Measuring electromotive force and terminal voltage of a cell

Set up a circuit consisting of a dry cell, a bulb, a switch and a voltmeter. Connect the voltmeter as shown in Figure 3.44.

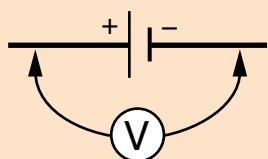


Figure 3.44

Take the reading in the voltmeter while the switch is on.

Then another reading while the switch is off.

Compare the two readings.

Which one is larger? Which reading is the e.m.f.?

Discuss with your group.

requires that you change only one thing at a time, which means that you must not change the resistance of the circuit.

There are two problems here:

1. Adding another cell adds a bit more resistance to the circuit.
All you can do about that is to choose cells that have a very low internal resistance (such as the lead–acid cells that make up a car battery) or to use a circuit for which the resistance is so high that another ohm or two makes virtually no difference.
2. The resistance of a metal wire changes as it heats up. This rules out circuits containing light bulbs, because as you add more batteries their temperature changes from less than red-hot to strongly white-hot: a change of perhaps 1000 degrees!

The difference between e.m.f. and terminal p.d. of a cell

A high resistance voltmeter connected across the terminals of a cell can give a good reading of the cell's e.m.f. However, if the cell is then connected in a circuit where the current is high, the reading on the voltmeter drops. This drop is caused by the cell's internal resistance.

Figure 3.46 shows the cell from Figure 3.45 connected in a circuit.

If the e.m.f. of this cell is E and it is connected to a resistor R , the potential difference across its terminals (V) will now be less than E because some of its energy is used to drive the current (I) through the internal resistance (r) in the cell. If R decreases, the current (I) increases and the terminal p.d. (V) of the cell will decrease.

The relationship between e.m.f., current and internal resistance in a circuit

The relationship between a cell's e.m.f. (E) and internal resistance (r) and the current (I) and resistance (R) in a circuit is given by the expression:

$$E = \text{p.d. across } R + \text{p.d. across } r$$

$$E = IR + Ir$$

$$E = I(R + r)$$

Worked example 3.15

In the circuit shown in Figure 3.46, the cell's e.m.f. is 10 V and its internal resistance is 2 Ω . Find the p.d. across the terminals of the cell when it is connected to a 3 Ω resistor.

First find the current in the circuit:

$$E = I(R + r)$$

$$I = \frac{E}{(R + r)}$$

$$= \frac{10}{(3 + 2)} = 2 \text{ A}$$

The p.d. across the cell terminals is equal to the p.d. across the resistor (3 Ω). Find the p.d. across the resistor:

$$V = IR = 2 \times 3 = 6 \text{ V}$$

Therefore the p.d. across the terminals of the cell is 6 V.

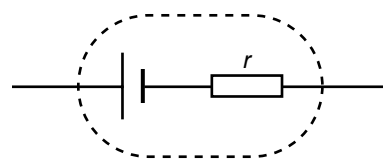


Figure 3.45 Representation of a cell indicating its internal resistance r .

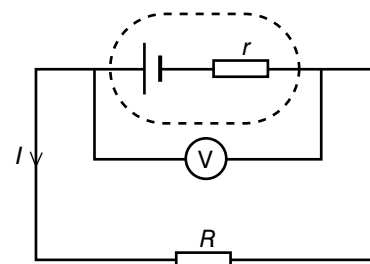
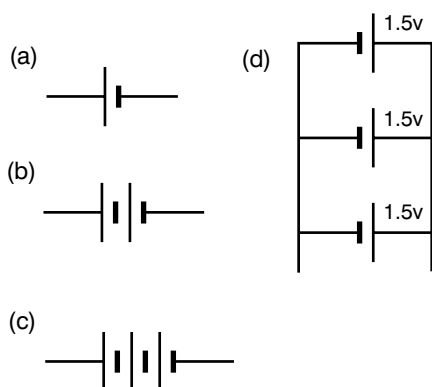


Figure 3.46

As $V = IR$ (Ohm's law) this can also be written as $E = V + Ir$, or $V = E - Ir$.



Combinations of cells in series and parallel

In Figure 3.47, (a), (b) and (c) show cells connected in series, (d) shows cells connected in parallel. Each cell has an e.m.f. of 1.5 V.

The total e.m.f. of cells connected in series is the sum of the e.m.f. of each cell: $E = E_1 + E_2 + \dots + E_n$. Equipment requiring high power uses cells arranged in series.

The e.m.f. of cells connected in parallel is the e.m.f. of an individual cell: $E = E_1 = E_2 = \dots = E_n$. The current supplied by a parallel arrangement of cells can be maintained for far longer than that supplied by a single cell, so equipment requiring a steady current for long periods will use cells arranged in parallel.

(a) 1 cell	total e.m.f. = 1.5 V
(b) 2 cells in series	total e.m.f. = 1.5 + 1.5 = 3 V
(c) 3 cells in series	total e.m.f. = 1.5 + 1.5 + 1.5 = 4.5 V
(d) 3 cells connected in parallel	total e.m.f. = 1.5 V

Figure 3.47 Combining cells.

Activity 3.8: Connecting cells together

For this activity you can use either the fruit cells made in Activity 3.3 or purchased 1.5 V cells. Connect a small light bulb in a circuit with one cell. Observe its brightness. Now take another cell and connect it in the circuit as shown in Figure 3.48a. Observe the brightness of the bulb. You can add further cells in this way if you wish.

The cells in Figure 3.48a are connected in series. Now connect two cells together in parallel, as shown in Figure 3.48b. Now connect these cells to a light bulb as shown in Figure 3.48c. Observe its brightness.

How does the brightness of the bulb in the circuit with one cell compare with its brightness in the circuit with two cells in series?

How does the brightness of the bulb in the circuit with the cells in parallel compare to the brightness when the cells are in series?

You can see that the cells are connected negative to negative, positive to positive in Figure 3.48b. See what happens if you connect them negative to positive and positive to negative before connecting them in a circuit. What do you find?

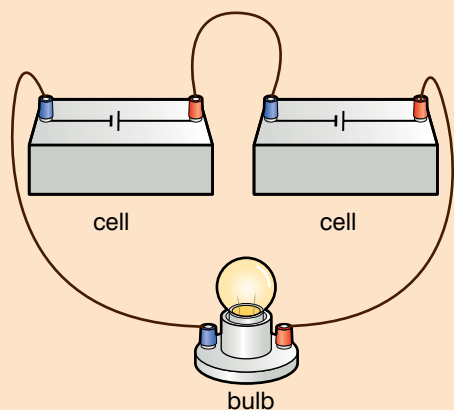


Figure 3.48a

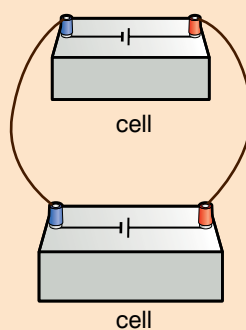


Figure 3.48b

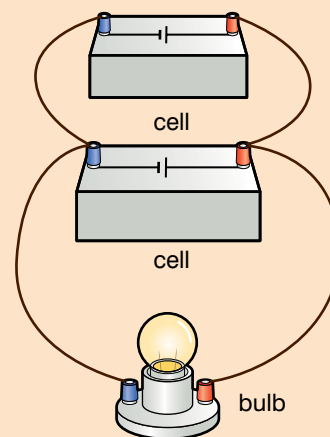


Figure 3.48c

Review questions

- Imagine you have a brother two years younger than you are. Explain to him why:
 - the '+' terminal of one cell in a circuit should be connected to the '-' terminal of a second cell, but
 - the '+' terminal of an ammeter needs to be connected to the '+' terminal of the battery.
- A circuit consists of three resistors in series: one of $3\ \Omega$, one of $7\ \Omega$ and one of $10\ \Omega$. A battery of e.m.f. $12\ \text{V}$ and negligible internal resistance is connected to the circuit. What size current will it supply?
- A battery has an e.m.f. of $3\ \text{V}$ and an internal resistance of $1\ \Omega$. What current will it give if it is connected to a circuit of resistance:
 - $2\ \Omega$
 - $4\ \Omega$?

What current will it give if its terminals are short-circuited?

- In Figure 3.49, A and B represent the terminals of a battery of e.m.f. $4\ \text{V}$ and internal resistance $0.5\ \Omega$. R is the total resistance of the circuit to which it is connected.
 - Explain why a voltmeter connected as shown across R would read the same as a voltmeter connected across the terminals of the battery.
 - Calculate the current which flows round the circuit if the resistance of R is $1.5\ \Omega$.
 - What then is the p.d. between the terminals of the battery?

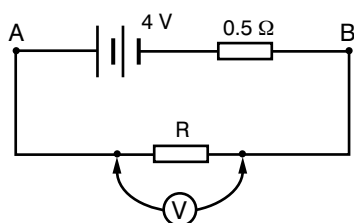


Figure 3.49

- What current is flowing through the $3\ \Omega$ resistor in Figure 3.50?
 - If its internal resistance is $2\ \Omega$, what is the e.m.f. of the battery?

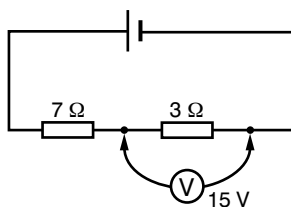


Figure 3.50

3.5 Electric energy and power

By the end of this section you should be able to:

- Define electrical energy and power in an electrical circuit.
- Find the relationship between KWh and joule.
- Use $P = VI = \frac{V^2}{R} = I^2R$ to solve problems in electric circuits.
- Use $W = VIt = I^2Rt = \frac{V^2t}{R}$ to calculate electric energy dissipated in an electric circuit.
- Calculate the cost of electrical energy expressed in KWh.

Worked example 3.16

A 60 W light bulb is switched on for 2 minutes. How many joules of electrical energy does it convert into heat and light in that time?

The power of the light bulb is 60 W; this means that it uses 60 J of energy every second.

It is switched on for
2 minutes = 120 s.

Energy = VIt (and Power = VI)

Total energy supplied =
 $60 \text{ J/s} \times 120 \text{ s} = 7200 \text{ J}$

Definitions for electrical energy and power

The word 'power' here means the rate at which energy is being supplied or converted. Power is measured in the units of joules per second (J/s), or watts (W).

The electrical energy produced by a current of I amperes flowing through a p.d. of V volts for a time t seconds is given by:

Energy = VIt J.

Power is the energy produced by an electrical appliance in one second. Thus, if $t = 1$:

Power = $VI \times 1 = VI$ W.

Worked example 3.17

If the potential difference across a working electrical motor is 50 V and the current is 2 A, calculate the power of the motor.

Power = $VI = 50 \times 2 = 100 \text{ W}$

Consider a resistor that converts all the electrical energy supplied to heat.

The power produced = VI

This can be rewritten (using Ohm's law, $V = IR$) as $IR \times I = I^2R$

Therefore, for a resistor

heat produced per second = power = I^2R .

If V and R are known, then the equation can be written (again using Ohm's law)

power = $VI = V \times \frac{V}{R} = \frac{V^2}{R}$.

Worked example 3.18

Two heating coils A and B produce heat at a rate of 1 kW and 2 kW, respectively, when connected to 250 V mains.

- Calculate the resistance of each resistor.
- Find the power they would produce when connected in series to the mains.

a) For the first resistor:

$$\text{Power} = VI$$

$$1000 = 250 \times I, \text{ so } I = \frac{1000}{250} = 4 \text{ A.}$$

$$\text{So } R = \frac{V}{I} = \frac{250}{4} = 62.5 \, \Omega$$

For the second resistor:

$$\text{Power} = VI$$

$$2000 = 250 \times I, \text{ so } I = \frac{2000}{250} = 8 \text{ A.}$$

$$\text{So } R = \frac{V}{I} = \frac{250}{8} = 31.25 \, \Omega$$

- b) If the resistors are wired in series, their total resistance is

$$R_1 + R_2 = 62.5 + 31.25 = 93.75 \, \Omega.$$

$$\text{using } R = \frac{V}{I}$$

$$93.75 = \frac{250}{I}, \text{ giving } I = \frac{250}{93.75}$$

$$= 2.67 \text{ A.}$$

$$\text{So power} = VI = 250 \times 2.67 = 666 \text{ W}$$

Notice that this is less than the power produced when either resistor is connected to the mains separately. Can you see why?

Worked example 3.19

Calculate the power of a water heater that draws a current of 10 A from a 220 V supply.

$$\text{Power} = VI = 10 \times 220 = 2.2 \text{ kW}$$

Cost of electrical energy

Electricity is distributed to homes and businesses and the quantity supplied is measured in kilowatt hours (KWh). A kilowatt hour is the energy used by a 1 kW appliance working for 1 hour. Consumers of electricity are charged for each kWh used.

How many joules are there in 1 kWh?

One kWh = energy transformed by 1 kW (1000 W) for 1 hour (3600 s) = energy when 1000 J are transformed each second for 3600 s = $1000 \times 3600 = 3\,600\,000 \text{ J} = 3.6 \text{ MJ}$

Worked example 3.20

How many units (kWh) are used by:

- a 3 kW electric fire used for 2 hours,
- a 100 W light bulb used for 15 hours?

a) Number of units = $3 \times 2 = 6 \text{ kWh}$

b) Number of units = $0.1 \times 15 = 1.5 \text{ kWh}$

Activity 3.9 Power requirements

Which appliances in your home consume the greatest amount of energy? Is it the refrigerator? The TV set? The electric stove? You can find out by inspecting appliances in your home and determining the number of watts each consumes and multiplying this by the number of hours operated. By law, each electric device must specify power requirements, and these are generally recorded on a small tag located on the appliance or on the power cable connected to it. Inspect all the appliances in your home and record power requirements in a table.

Worked example 3.21

A 1.5 kW electric fire is accidentally left on overnight for eight hours. The cost of a unit of electrical energy is 0.273 Birrs. How much money has been wasted?

The number of units (kWh) = power in kW $\times$ time in hours =
 $1.5 \times 8 = 12 \text{ kWh}$

The total cost = 12 $\times$ cost of each unit = $12 \times 0.273 = 3.28$ Birrs

Worked example 3.22

A current of 4 A flows through an electric fire for 1 hour. The supply voltage is 240 V. What energy is transformed by the fire in 1 hour? (1 hour = 60 $\times$ 60 seconds)

$W = VIt = 240 \times 4 \times 3600 = 3.456 \text{ megajoules (MJ)}$

Worked example 3.23

The Ethiopian Power Corporation is distributing power saving lamps to the public free of cost. If an 11 W (0.011 kW) power saving bulb is used in place of the equivalent 60 W (0.06 kW) conventional lamp, and each type of bulb is used for 10 hours a day for four weeks (seven days a week), how much would the customer of the Ethiopian Power Corporation save?

Number of hours = $10 \times 7 \times 4 = 280$

Cost of one unit of electricity is 0.273 Birrs

Conventional lamp

Number of units (kWh) = power in kW $\times$ time in hours = $0.06 \times 280 = 16.8 \text{ kWh}$

Total cost of using conventional bulb = $16.8 \times 0.273 = 4.59$ Birrs

Power-saving lamp

Number of units = $0.011 \times 280 = 3.08 \text{ kWh}$

Total cost of using power-saving lamp = $3.08 \times 0.273 = 0.84$ Birrs

The customer would save 3.75 Birrs.

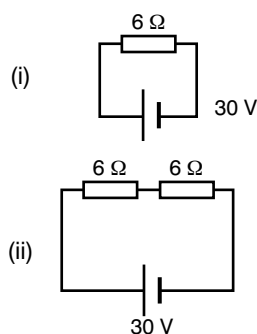


Figure 3.51

Review questions

1. Work out the sources of energy used at home. Can you suggest any economies or improvements?
2. Carry out the same analysis for your school.
3. In the circuit of Figure 3.51(i) calculate:
 - a) the current flowing
 - b) the number of joules per second (power) of heat produced in the 6 Ω resistor.

Another $6\ \Omega$ resistor is added in series with the first (Figure 3.51(ii)). Calculate:

- c) the current flowing now
- d) the power produced in the two $6\ \Omega$ resistors together
- e) the power produced in a single $6\ \Omega$ resistor.

Comment on the reasons for the difference in the heat produced in a $6\ \Omega$ resistor in the two circuits.

3.6 Electric installation and safety rules

By the end of this section you should be able to:

- Understand the dangers of mains electricity.
- Have some awareness of safety features incorporated in mains electrical installations.
- Understand the nature of the generation and supply of electricity in Ethiopia.
- Consider employment prospects in Ethiopia's electricity industry.

KEY WORDS

fuse *protective device for protecting an electrical circuit, containing a wire that melts and breaks when the current exceeds a certain value*

In Ethiopia, while traditional sources of energy, such as wood, are still of great importance, the provision of mains electricity is becoming increasingly significant. Mains electricity is generated, transmitted and distributed by the Ethiopian Electric Power Corporation (EEPCO).

While Ethiopia uses some fossil fuels for electricity generation, the country is fortunate in having access to sources of hydroelectric power at Melka Wakena, Finchaa, Koka, Awash, Tis Abay, Gilgel Gibe and other planned sites, for electricity generation. Ethiopia also has potential for using other renewable energy sources, such as solar, wind and geothermal energy, for generating electricity.

Work is underway to increase the availability of mains electricity in Ethiopia. Mains electricity is much more powerful than the electricity we have studied in this unit, and great care must be taken when using it. Any work being undertaken on mains electrical installations must only be undertaken by those who have qualified through the Electrical Professional Competence Certification Scheme (operated by the Ethiopian Electricity Agency).

Various safety features are incorporated into mains electrical installations; one of these features is the **fuse**, which uses the heating effect studied earlier in this unit. A fuse is a small, thin piece of wire in a glass tube that has metal connectors on each end. It is designed to be used in a circuit to allow a current of a certain size to pass through it. If the current in the circuit increases beyond this point, the fuse heats up and then breaks, causing the current in the circuit to stop. The fuse is a very useful safety device, protecting appliances from surges of current.

KEY WORDS

earthed *circuit connected to the earth, allowing any dangerously large current to be safely discharged*

Electrical safety is also increased if appliances are **earthed**. This means that if there were a fault in an appliance, any dangerous large current would run harmlessly to the earth and cause the fuse to blow, thus preventing injury.

Miniature circuit breakers (MCBs) can now be used in place of fuses in some appliances. They cut off the power if there is a fault that causes the appliance to overheat. They can be re-set when the appliance cools down.

Activity 3.10: Electrical safety

Take five 1.5 V cells, a 2 A fuse and a selection of connecting wires. Connect the circuit shown in Figure 3.52.

Look carefully at the fuse, and while observing the fuse closely, close the switch.

The fuse consists of a very fine length of wire inside a transparent tube with metal connectors at each end.

When the switch is closed, the wire in the tube quickly becomes red and then separates into two pieces. The wire becomes dark again as it cools down. At the end of the experiment, the wire in the fuse has a gap in the centre.

There might be a small lump of metal on one of

the broken ends of wire. This is a result of the metal becoming very hot and melting as the current passes through it, before the fuse wire breaks and causes the current to stop.

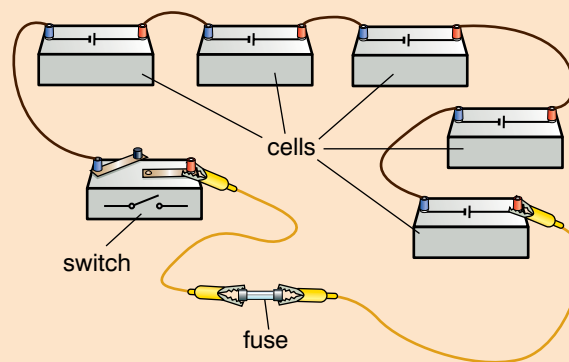


Figure 3.52

Engineering project

New designs for cars employ hybrid motors. Cars use both electricity and gasoline for power. When going down hill, instead of standard brakes, which convert kinetic energy into heat, these cars capture energy by using 'electric brakes' that recharge the battery or convert the energy into rotational energy in a massive fly wheel.

Try designing on paper a hybrid car that captures as much energy as possible. In teams you may wish to build a model of your design, showing innovative ways to save energy.

Earth circuit leakage breakers (ECLBs) are used in place of fuses in domestic electricity supply boards, where they are useful in cutting off the supply very quickly if a fault occurs.

Small, plug-sized ECLBs (also called residual current circuit breakers) are also useful in preventing injury when using electrical equipment outdoors. They will cut off the power supply very quickly if the power cable is cut accidentally.

Review questions

- For safety, the circuit for a 3 kW metal-bodied electric kettle is both earthed and contains a fuse.
 - Why is the kettle earthed?
 - Why does the circuit contain a fuse?
- In a house you might find i) MCBs, ii) an ECLB.
 - Say where each of these may be found.
 - Explain the purpose that each might be serving.

End of unit questions

1. If 36 C of charge pass through a wire in 4 s, what current is it carrying?
2. An appliance has a resistance of $5\ \Omega$ and requires a current of 0.5 A for it to work. The only battery available has an e.m.f. of 12 V and negligible internal resistance. What extra resistance will you need to provide to limit the current to its correct value. Draw the circuit you would assemble.

3. A student incorrectly set up the circuit shown in Figure 3.53.
 - a) What is wrong with it?
 - b) Assume that both the ammeter and the battery have negligible resistance. Starting from one terminal of the battery, what route will all the current take as it flows round the circuit? How much resistance will this circuit have? What will the ammeter read?
 - c) What will the voltmeter read?
 - d) The student then reconnects the circuit so that the two resistors, the ammeter and the voltmeter are all in series with the battery. What would you expect the ammeter to read now? Give a reason for your answer.

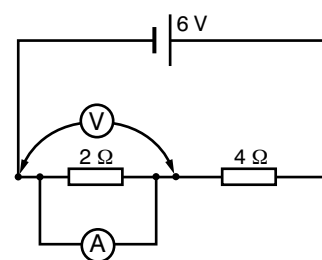


Figure 3.53

4. A 12 V light bulb is put under test, with the following results:

Current through bulb (A)	0.4	0.6	0.8	1.0	1.2	1.4	1.6
p.d. across bulb (V)	1.4	2.6	3.9	5.5	7.4	9.7	12.6

- a) Draw the circuit you would use to obtain these readings.
 - b) Copy the table out, and for each set of readings work out the resistance R of the bulb.
 - c) Plot a graph of R against I . Why is it not valid to assume the graph goes through the origin?
 - d) Use your graph to estimate the resistance of the bulb at room temperature. (Hint: in what circumstance will the bulb not be heated at all?)
5. Constantan is the name of an alloy that is sometimes used for making resistors in the laboratory. Its resistivity is $4.9 \times 10^{-7}\ \Omega\ \text{m}$. Calculate the resistance of a 3 m long constantan wire with $1\ \text{mm}^2$ cross-sectional area.
 6. Draw a labelled diagram of a dry cell. State the function of each of its parts.
 7. The resistivity of copper is $2 \times 10^{-8}\ \Omega\ \text{m}$. Work out the resistance of a copper wire of $1\ \text{mm}^2$ cross-sectional area and 3 m long.

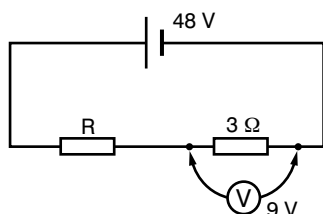


Figure 3.54

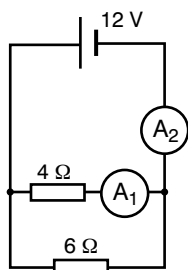


Figure 3.55

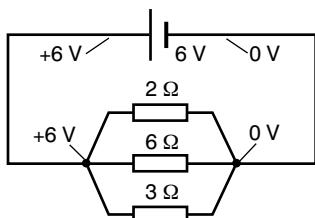


Figure 3.56

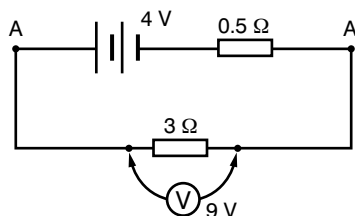


Figure 3.57

8. A battery sends a current of 3 A through a 4 Ω resistor.
 - a) What must the e.m.f. of the battery be?
 - b) How many coulombs of charge will pass through the resistor in 1 minute?
9. Explain why it is wrong to write the units of resistivity as ohm/metre (Ω/m) with the stroke between the two quantities.
10. Draw a circuit diagram to show a battery of two cells and three light bulbs connected in series. Include a switch to turn the lights on and off. Does it matter where the switch is placed in the circuit?
11. a) What current is flowing round the circuit in Figure 3.54?
b) How great must the p.d. be across resistor R?
c) What is the resistance of resistor R?
12. What should be the reading of ammeters A₁ and A₂ in Figure 3.55?
13. These questions refer to the circuit drawn in Figure 3.56.
 - a) What are the currents through the 6 Ω branch, through the 3 Ω branch and through the 2 Ω branch?
 - b) What is the total current drawn from the battery?
 - c) As far as the battery is concerned, what is the total resistance of its whole circuit?
 - d) Compare this with the value obtained from
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
14. In Figure 3.57, A and B represent the terminals of a battery of e.m.f. 4 V and internal resistance 0.5 Ω. R is the total resistance of the circuit to which it is connected. Calculate the current that flows round the circuit and the p.d. between the terminals of the battery if the resistance R is
 - a) 7.5 Ω
 - b) 0 Ω.